THE MERCHANTS AND MECHANICS COMMERCIAL ARITHMETIC.

WADE.

6. One thousand, and forty-two.
7. Thirty thousand, nine hundred and seven.
8. Forty-six thousand, and four hundred.
9. Ninety-two thousand, one hundred and eight.
10. Sixty-eight thousand, and seventy.
11. One hundred and twenty-four thousand, six hundred and thirty.
12. Two hundred thousand, one hundred and sixty.
13. Four hundred and five thousand, and forty-five.
14. Three hundred and forty thousand.
15. Nine hundred thousand, seven hundred and twenty.
16. One million, and seven hundred thousand.
17. Thirty-six millions, twenty thousand, one hundred and fifty.
18. One hundred millions, and forty-five.
19. Mercury is thirty-seven millions of miles from the sun.
20. Venus, sixty-nine millions.
21. The Earth, ninety-five millions.
22. Mars, one hundred and forty-five millions.
23. Jupiter, four hundred and ninety-four millions.
24. Saturn, nine hundred and seven millions.
25. Herschel, one billion, eight hundred and ten millions.
26. Seven billions, nine hundred millions, and forty thousand.
27. Sixty billions, seven millions, and four hundred.
28. One hundred and thirteen billions, six hundred and fifty thousand.
29. Four hundred and six billions, eighty millions, and seven hundred.
30. Twenty-five trillions, and ten thousand.
31. Two hundred and six billions, five hundred and sixty thousand, and forty-five.
32. Six hundred millions, seventeen thousand, three hundred and eight.
33. Ninety-seven trillions, sixteen millions, seventy thousand, and thirty.
34. Eight hundred and forty billions, fifty millions, three hundred and one thousand.
35. Three hundred and sixty-five quadrillions, tw

THE

MERCHANTS & MECHANICS' COMMERCIAL ARITHMETIC;

OR,

INSTANTANEOUS METHOD OF COMPUTING NUMBERS.

By JOHN E. WADE.

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PREFACE.

Mathematical laws are the acknowledged basis of all science. Ever since the streets of Athens resounded with that historical cry of "Eureka," emanating from one of antiquity's greatest mathematicians, the science has been steadily progressing.

It is not our purpose in this small work to introduce any of the higher branches of mathematics, viz., Algebra, Conic Sections, Calculus, etc. Our object is merely to present to the public a system of calculation that is practical to every business man. It consists of the addition of numbers on a principle entirely different from the one ordinarily used. In the practical application of this new principle of Addition scarcely any mental labor is required, compared with the principle of Addition set forth in standard works. The superiority we claim for this principle above all others is this—that it requires no great mental exertion, affording the greatest facilities to the calculator in the addition of numbers, enabling him to add a whole day without any mental fatigue; whereas, by the ordinary way, it is very laborious and fatiguing.

Our system of calculation also embraces a concise, rapid, and, at the same time, practical method of Multiplication, by which one is enabled to arrive at the product of any number of figures, multiplied by any number, immediately, without the use of partial products.

This small work also embraces the shortest and most concise method for the computation of Interest ever introduced to the public. Our system for computing interest is entirely different from any rule ever introduced for the computation of either Simple or Compound Interest. A student, having gone no further than Long Division in Arithmetic, can, by our rule, calculate Simple or Compound Interest at any given rate per cent., for any given time, in one tenth of the time that the best calculators will compute it by the rules laid down in other books. By using our rules you can entirely avoid the use of fractions, and save the calculation of 75 to 100 figures where years, months and days are given on a note.

INTRODUCTION.

Quantity is that which can be increased or diminished by augments or abatements of homogeneous parts. Quantities are of two essential kinds, *Geometrical* and *Physical.*

1. *Geometrical* quantities are those which occupy space—as *lines, surfaces, solids, liquids, gases,* etc.

2. *Physical* quantities are those which exist in the time, but occupy no space ; they are known by their character and action upon geometrical quantities, as *attraction, light, heat, electricity* and *magnetism, colors, force, power,* etc.

To obtain the magnitude of a quantity we compare it with a part of the same ; this part is imprinted in our mind as a *unit,* by which the whole is measured and conceived. No quantity can be measured by a quantity of another kind, but any quantity can be compared with any other quantity, and by such comparison arises what we call *calculation or Mathematics.*

MATHEMATICS.

Mathematics is a science by which the comparative value of quantities is investigated ; it is divided into—

1. ARITHMETIC, that branch of Mathematics which treats of the nature and property of numbers. It is subdivided into *Addition*, *Subtraction*, *Multiplication*, *Division*, *Involution*, *Evolution* and *Logarithms*.

2. ALGEBRA, that branch of Mathematics which employs letters to represent quantities, and by that means performs solutions without knowing or noticing the *value* of the quantities. The subdivisions of Algebra are the same as in Arithmetic.

3. GEOMETRY, that branch of Mathematics which investigates the relative property of quantities that occupies space; its subdivisions are *Longemetry*, *Planemetry*, *Stereometry*, *Trigonometry* and *Conic Sections*.

4. DIFFERENTIAL-CALCULUS, that branch of Mathematics which ascertains the mean effect produced by group of continued variable causes.

5. INTEGRAL-CALCULUS, the contrary of Differential, or that branch of Mathematics which investigates the nature of a continued variable cause that has produced a known effect.

NOTATION

is the rule for writing numbers. The Arabic characters or figures are 1, 2, 3, 4, 5, 6, 7, 8, 9, 0. These figures have two values, viz., the *simple value*—its value when taken alone—and the *local value*—its value when used with another figure or figures. Counting from the right hand the first figure expresses *units*, the second *tens*, the third *hundreds, and every removal of a figure one place towards the left increases its local value ten times.*

NUMERATION

is the rule for reading numbers. According to the French method—the one generally in use—every three figures constitute a period. Commencing upon the right, the periods are successively named units, thousands, millions, billions, trillions, quadrillions, quintillions, sextillions, septillions, octillions, nonillions, decillions, undecillions, duodecillions, tredecillions, quartodecillions, quindecillions, sexdecillions, septdecillions, octodecillions, nondecillions, virgitillions. By committing the names of these periods to memory you will be enabled to enumerate sixty-six figures—a number far greater than will ever be required in practice. Beginning at the left hand, read each period separately, giving the name to each period except the last, or period of units.

EXAMPLE.	37,	546,	829,	106,	397,	463,	512,	460,	837
	Septillions.	Sextillions.	Quintillions.	Quadrillions.	Trillions.	Billions.	Millions.	Thousands.	Units.

Read—thirty-seven septillions, five hundred and forty-six sextillions, eight hundred and twenty-nine quintillions, etc.

SIGNS USED IN MATHEMATICS.

The sign $=$ is called the sign of *equality*. When placed between two numbers it signifies that they are equal to each other.

The sign $+$ is called *plus*, which signifies *more*. When placed between two numbers it denotes that they are to be added together.

The sign — is called *minus*, which signifies *less*. When placed between two numbers it denotes that the one after it is to be taken from the one before it.

The sign $\times$, when placed between two numbers, denotes that they are to be multiplied together.

The sign $\div$, placed between two numbers, denotes that the number before it is to be divided by the number after it. Division is also expressed by writing the *dividend* above a short horizontal line and the divisor below it, thus : $\frac{\text{Dividend.}}{\text{Divisor.}}$

The sign $, contraction of U. S., denotes dollars, or United States currency.

The sign $\surd$ is called the radical sign, which, placed before a number, denotes that its square root is to be extracted.

ADDITION

is uniting two or more numbers to find their *sum* or amount.

Addition is the basis of all numerical calculations, and is used in all departments of business. To be able to add with ease, precision and rapidity, is a great desideratum by every accountant. To aid the business man in acquiring facility and accuracy in adding numbers the following methods are presented as among the best. But it may as well be understood that no method can be presented that will entirely do away with all labor of the brain—for to add accurately and rapidly can only be acquired by close application and careful, constant practice. In writing down the numbers for addition, those of the same order should be written *exactly* under each other. Too much care cannot be used in this respect. Many serious blunders are often made from no other cause than the bad habit of writing numbers carelessly, getting units under tens or tens under units, etc. Perhaps, too, this would be as proper a time as any to urge the importance of *making plain figures.* If letters are badly made you can judge from such as are known, but if one figure be illegible it cannot be inferred from the others. Making figures is a habit ; therefore, a little care at first in making them *plain*, may save much anxious toil and serious trouble afterward. After having written the numbers, those of the same order *exactly* under each other, begin with the right hand, or unit column, and proceed to add together all the figures in that column, and write down the *entire* amount underneath. Then carry or add all except the unit figure of that amount to the next column, adding up that column as before, and write the entire amount underneath the former one. Proceed in this manner with each column until all are added, setting down the entire amount of each column, one under the other. The last amount, with the unit figure of each amount annexed, will be the correct amount of the entire sum.

EXAMPLE.	
3462	
895	
7604	
9346	
8753	
4927	
2861	
920	
803	
6746	
25	
7934	
46	am't of units.
47	" " tens.
82	" " hund.
54	" " thou.
54276	entire amount.

Process.—Commencing at 4 in the unit column, say 4, 9, 15, 18, 19, 26, 29, 35, 39, 44, 46, setting down the entire amount. Then add the four tens of this amount to to the columns of tens thus: 4, 7, 9, 13, 15, 21, 23, 28, 32, 41, 47, setting down the amount. Then carry the 4 of this amount to the next column, and say 4, 13, 20, 28, 37, 45, 54, 61, 64, 70, 78, 82, setting down the amount as before. Then carrying the 8 to the next column, say 8, 15, 21, 23, 27, 35, 44, 51, 54. Then this last amount, with the unit figure of each amount annexed in the order as they occur, reading upward, will be the entire amount. The partial amounts can be set down upon a separate piece of paper, if desired, and only the entire amount written under the sum. The practice, however, of setting down the amount of each column will be found highly convenient—as, knowing what number you carried, you will be able at any time to add any column without being obliged to add all that precede it.

THE EASY WAY TO ADD.

EXAMPLE 2.—EXPLANATION.

Commence at 9 to add, and add as near 20 as possible, thus: $9+2+4+3=18$; place the 8 to the right of the 3, as in example; commence at 6 to add $6+4+8=18$; place the 8 to the right of the 8, as in example; commence at 6 to add $6+4+7=17$; place the 7 to the right of the 7, as in example;

commence at 4 to add 4+9+3=16; place the 6 to the right of the 3, as in example; commence at 6 to add 6+4+7+17; place the 7 to the right of the 7, as in example. Now, having arrived at the top of the column, we add the figures in the new column thus: 7+6+7+8+8=36; place the right hand figure of 36, which is a 6, under the original column, as in example, and add the left hand figure, which is a 3, to the number of figures in the new column; there are 5 figures in the new column, therefore 3+5=8; prefix the 8 with the 6 under the original column, as in example; this makes 86, which is the sum of the column.

REMARK.—If, upon arriving at the top of the column, there should be one, two or three figures whose sum will not equal 10, add them on to the sum of the figures of the new column, never placing an extra figure to the new column unless it be an excess of units over 10.

7^7
4
6
3^6
9
4
7^7
4
6
8^8
4
6
3^8
4
2
9
—
86

EXAMPLE.—EXPLANATION.

2+6+7=15, drop 10, place the 5 to the right of the 7; 6+5+4=15, drop 10, place the 5 to the right of the 4, as in example; 8+3+7=18, drop 10, place the 8 to the right of the 7, as in example; now we have an extra figure, which is 4; add this four to the top figure of the new column, and this sum on the balance of the figures in the new column, thus: 4+8+5+5=22; place the right hand figure of 22 under the original column, as in example, and add the left hand figure of 22 to the number of figures in the new column, which are three, thus: 2+3=5; prefix this five to the figure 2, under the original column; this makes 52, which is the sum of the column.

4
7^8
3
8
4^5
5
6
7^5
6
2
—
52

ANOTHER METHOD.

$$\begin{array}{r} 6\,4 \\ 2^{7}8^{6} \\ 3\,2 \\ 5\,6 \\ 3\,7^{6} \\ 4\,9 \\ 1^{7}7^{6} \\ 2\,3 \\ 6\,2 \\ 3\,4 \\ \hline 402 \end{array}$$

Process.—Commence at 4 to add, and add as near 20 as possible, thus: 4+2+3+7=16; place the 6 to the right hand of the 7, as in example. Commence at 9 to add, 9+7=16; place the 6 to the right of the 7. Commence at 6 to add, 6+2+=8 16; place the 6 to the right of the 8, as in example. Now, having arrived at the top of the column, or so near that the remaining figures will not equal 10, we add the figures in the new column, thus: 6+6+6=18+4 (the remaining figure)=22. Set the unit figure under the column, and add the left hand figure, which is 2, to the number of figures in the new column. The number of figures in the new column are 3, therefore 2+3=5. Carry the 5 to the 3 in the next column, and say, 5+3+6+2=1=17; place the 7 to the right of the 1, as in example. Commence at 4 to add, 4+3+5+3+2=17; place the 7 to the right of the 2. Now, having arrived at the top of the column, or so near that the remaining figures will not equal 10, we add the figures in the new column, 7+7=14+6 (the remaining figure)=20. Set the right hand figure, which is 0, under the column, and add the left hand figure, 2, to the number of figures in the new column, which are 2; therefore 2+2=4, which prefix to the amounts already set down, and you have 402, the entire amount.

This method is useful only in adding very long columns of figures, say a long ledger column, where the footings of each column would be two or three hundred, in which case it is more easy than any other mode, as the mind is relieved at intervals, and the mental labor of retaining the whole amount as you add is avoided, which is very important to any person whose mind is constantly employed in various commercial calculations. The small figures in the new column can be made lightly in pencil, so as to be easily erased, and the ledger will only show the total amounts.

But, whatever method may be used, never for *once* allow yourself to add in this way : 4 and 2 are 6, and 3 are 9, and 7 are 16, and 9 are 25, and 7 are 32, etc. It is just as easy to name the results of two figures at once, and four times as rapid, thus : 4, 6, 9, 16, 25, 32, 38, 40, 48, 52 ; or, better still, to form the habit of grouping them, thus : 6, 16, 25, 32, 52. We see at a glance that 4 and 2 are 6, that 3 and 7 are 10, making 16, and 9 are 25, and 7 are 32 ; that 6 and 4 are 10, and that 2 and 8 are 10, making 20, which added to 32 are 52. By practicing this method a short time you will soon acquire a proficiency that will astonish the uninitiated.

The method of proof most commonly employed is to reverse the process of addition, or commence at the top and add downward. If the results agree the work is supposed to be right.

MULTIPLICATION.

Multiplication, in its most general sense, is a series of additions of the same number; therefore, in Multiplication, a number is repeated a certain number of times, and the result thus obtained is called the product. When the multiplicand and the multiplier are each composed of only two figures, to ascertain the product we have the following

Rule.—*Set down the smaller factor under the larger, units under units, tens under tens. Begin with the unit figure of the multiplier, multiply by it first the* *units of the multiplicand, setting the units of the product, and reserving the tens to be added to the next product; now multiply the tens of the multiplicand by the unit figure of the multiplier, and the units of the multiplicand by tens figure of the multiplier; add these two products together, setting down the units of their sum, and reserving the tens to be added to the next product; now multiply the tens of the multiplicand by the tens figure of the multiplier, and set down the whole amount. This will be the complete product.*

Remark.—Always add in the tens that are reserved as soon as you form the first product.

EXAMPLE 1.—EXPLANATION.

$$\begin{array}{r} 24 \\ 31 \\ \hline 744 \end{array}$$

1. Multiply the units of the multiplicand by the unit figure of the multiplier, thus: 1×4 is 4; set the 4 down as in example. 2. Multiply the tens in the multiplicand by the unit figure in the multiplier, and the units in the multiplicand by the tens figure in the multiplier, thus: 1×2 is 2, 3×4 are 12; add these two products together, $2+12$ are 14; set the 4 down as in example, and reserve the 1 to be added to the next product. 3. Multiply the tens in the multiplicand by the tens figure in the multiplier, and add in the tens that were reserved, thus: 3×2 are 6, and $6+1=7$; now set down the whole amount, which is 7.

EXAMPLE 2.—EXPLANATION.

$$\begin{array}{r} 53 \\ 84 \\ \hline 4452 \end{array}$$

1. Multiply units by units, thus: 4×3 are 12; set down the 2 and reserve the 1 to carry. 2. Multiply tens by units and units by tens, and add in the 1 to carry on the first product, then add these two products together, thus: 4×5 are $20+1$ are 21, and 8×3 are 24, and $21+24$ are 45; set down the 5 and reserve the 4 to carry to

the next product. 3. Multiply tens by tens, and add in what was reserved to carry, thus: 8×5 are $40+4$ are 44; now set down the whole amount, which is 44.

EXAMPLE 3.—EXPLANATION.

5×3 are 15, set down the 5 and carry the 1 to the next product; 5×4 are $20+1$ are 21; 2×3 are 6, $21+6$ are 27, set down the 7 and carry the 2; 2×4 are $8+2$ are 10; now set down the whole amount.	43 25 —— 1075

When the multiplicand is composed of three figures, and there are only two figures in the multiplier, we obtain the product by the following

RULE.—*Set down the smaller factor under the larger, units under units, tens under tens; now multiply the first upper figure by the unit figure of the multiplier, setting down the units of the product, and reserving the tens to be added to the next product; now multiply the second upper by units, and the first upper by tens, add these two products together, setting down the units figure of their sum, and reserving the tens to carry, as before; now multiply the third upper by units and the second upper by tens, add these two products together, setting down the units figure of their sum and reserving the tens to carry as usual; now multiply the third upper by tens, add in the reserved figure, if there is one, and set down the whole amount. This will be the complete product.*

REMARK.—One of the principal errors with the beginner in this system of multiplication is neglecting to add in the reserved figure. The student must bear in mind that the reserved figure is added on to the first product obtained after the setting down of a figure in the complete product.

Example 1.—Explanation.

Multiply first upper by units, 5×3 are 15, set down the 5, reserve the one to carry to the next product; now multiply second upper by units and first upper by tens, 5×2 are 10 $+1$ are 11, 4×3 are 12, add these products together $11+12$ are 23, set down the 3, reserve the 2 to carry; now multiply third upper by units and second upper by tens, add these two products together, always adding on the reserved figure to the first product: 5×1 are $5+2$ are 7, 4×2 are 8, and $7+8$ are 15, set down the 5, reserve the 1; now multiply third upper by tens, and set down the whole amount: 4×1 are $4+1$ are 5, set down the 5. This will give the complete product.

$$\begin{array}{r} 123 \\ 45 \\ \hline 5535 \end{array}$$

Multiply 123 by 456 in a single line.

Here the first and second places are found as before; for the third add 6×1, 5×2, 4×3, with the 2 you had to carry, making 30, set down 0 and carry 3, then drop the units' place and multiply the hundreds and tens crosswise, as you did the tens and units, and you find the thousand figure; then, dropping both units and tens, multiply the 4×1, adding the one you carried, and you have 5, which completes the product. The same principle may be extended to any number of places; but let each step be made perfectly familiar before advancing to another. Begin with two places, then take three, then four, but always practicing some time on each number, for any hesitation as you progress will confuse you.

$$\begin{array}{r} 123 \\ 456 \\ \hline 56088 \end{array}$$

N. B.—The following mode of multiplying numbers will only apply where the sum of the two last or unit figures equal ten, and the other figures in both factors are the same.

Useful and Interesting Contractions,

when the multiplier is 10, 100 or 1, with any number of ciphers annexed.

RULE.—*Annex as many ciphers to the multiplicand as there are ciphers in the multiplier; the number so formed will be the product required.*

EXAMPLE 1. Multiply 7854 by 10000.

There are *four* ciphers in the multiplier, therefore annex *four* ciphers to the multiplicand.

$7854 \times 10000 = 78540000$. Product.

To multiply any number composed of two figures by 11.

RULE.—*Write the sum of the figures between them.*

2. $45 \times 11 = 495$. Ans. $4+5=9$.

3. 62×11. $6+2=8$, placed between 6 and $2 =$ 682. Ans.

REMARK.—When the sum of the two figures is over 9 increase the left hand figure by the one to carry

4. 68×11. Here, $6+8=14$, over 9. Ans. 748.

To multiply by any number of 9s.

RULE.—*Annex as many ciphers to the multiplicand as there are* 9s *in the multiplier, and subtract the multiplicand.*

5. Multiply 6472 by 999.

Process.—$6472000 - 6472 = 6465528$. Ans.

To write down the square of any number of 9s at once without multiplying.

RULE.—*Write down as many* 9s, *less one, as there are* 9s *in the given number, an* 8, *as many* 0s *as* 9s, *and a* 1.

6. What is the square of 999? Ans. 998001.

Explanation.—As there are three 9s in the given number, we write down two 9s, then an 8, then two 0s and a 1.

7. Square 9999999. Ans. 99999980000001.

To multiply any number by two figures when the unit figure is 1.

RULE.—*Multiply by the ten's figure, and set the product under the ten's figure of the multiplicand; then add.*

8. Multiply 6357 by 41.

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  6357
 25428
 ------
 260637 Ans.
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REMARK.—If ciphers intervene, as 201, 40001, etc., multiply as before, but set the product as many places to the left of the tens as there are ciphers.

9. Multiply 48546 by 3001.

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     48546
 145638
 ---------
 145686546 Ans.
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To multiply when there are ciphers at the right of one or both factors.

RULE.—*Multiply the significant figures together, and annex to the product as many ciphers as there are ciphers on the right of both factors.*

Mental Operations in Fractions.

To square any number containing $\frac{1}{2}$, as $6\frac{1}{2}$, $9\frac{1}{2}$.

RULE.—*Multiply the whole number by the next higher whole number and annex $\frac{1}{4}$ to the product.*

EXAMPLE 1. What is the square of $7\frac{1}{2}$? Ans. $56\frac{1}{4}$.

We simply say 7 times 8 are 56, to which we add $\frac{1}{4}$

2. What will $9\frac{1}{2}$ lbs. beef cost at $9\frac{1}{2}$ cts. a lb.?
3. What will $12\frac{1}{2}$ yds. tape cost at $12\frac{1}{2}$ cts. a yd.?
4. What will $5\frac{1}{2}$ lbs. nails cost at $5\frac{1}{2}$ cts. a lb.?
5. What will $11\frac{1}{2}$ yds. tape cost at $11\frac{1}{2}$ cts. a yd.?
6. What will $19\frac{1}{2}$ bu. bran cost at $19\frac{1}{2}$ cts. a bu.?

REASON.—We multiply the whole number by the next higher whole number, because half of any number taken twice and added to its square is the same as to multiply the given number by *one* more than itself. The same principle will multiply any two *like* numbers together, when the sum of the fractions is *one*, as $8\frac{1}{3}$ by $8\frac{2}{3}$, or $11\frac{3}{8}$ by $11\frac{5}{8}$, etc. It is obvious that, to multiply any number by any two fractions whose sum is *one*, the sum of the products *must be the original number*, and adding the number to its square is simply to multiply it by *one* more than itself—for instance, to multiply $7\frac{1}{4}$ by $7\frac{3}{4}$ we simply say 7 times 8 are 56, and then, to complete the multiplication, we add, of course, the product of the fractions ($\frac{3}{4}$ times $\frac{1}{4}$ are $\frac{3}{16}$), making $56\frac{3}{16}$ the answer.

7. Multiply 356000 by 2400.

```
356000
  2400
------
1424
712
---------
854400000 Ans.
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To square any number ending in 5.

RULE.—*Omit the 5 and multiply the number as it will then stand by the next higher number, and annex 25 to the product.*

8. What is the square of 65 ? Ans. 4225.

Explanation.—We simply say 7 times 6 are 42, and annex 25.

9. What is the square of 85 ? Ans. 7225.

10. Square 295. Ans. 87025.

Explanation.—Multiply 29 by 30 and annex 25.

To square any number containing $\frac{1}{2}$.

RULE.—*Multiply the whole number by the next higher whole number and annex $\frac{1}{4}$ to the product.*

11. What is the square of $8\frac{1}{2}$? Ans. $72\frac{1}{4}$.

We simply say 9 times 8 are 72 and annex $\frac{1}{4}$.

12. What will $12\frac{1}{2}$ pounds beef come to at $12\frac{1}{2}$ cents a pound ? Ans. $1.56\frac{1}{4}$.

13. What will $6\frac{1}{2}$ pounds spike come to at $6\frac{1}{2}$ cents a pound ? Ans. $42\frac{1}{4}$.

To multiply any two *like* numbers together when the sum of the fractions is one.

RULE.—*Multiply the whole number by the next higher whole number, and to the product add the product of the fractions.*

REMARK.—To find the product of the fractions multiply the numerators together for a new numerator and the denominators for a new denominator.

14. Multiply $6\frac{2}{5}$ by $6\frac{3}{5}$. Ans. $42\frac{6}{25}$.

Explanation.—Multiply 6, the whole number, by 7, the next higher whole number=42. We then multiply the numerators of the fractions, $2\times3=6$, and the denominators, $5\times5=25$, making the product $\frac{6}{25}$, which we add to the product of the whole number, 42.

15. Multiply $7\frac{1}{3}$ by $7\frac{2}{3}$. Ans. $56\frac{2}{9}$.

16. Multiply $11\frac{3}{7}$ by $11\frac{4}{7}$, Ans. $132\frac{12}{49}$.

17. Multiply $29\frac{1}{3}$ by $29\frac{2}{3}$. Ans. $870\frac{2}{9}$.

To multiply any two like numbers together, each of which has a fraction with a like denominator, as $3\frac{3}{4}$ by $5\frac{1}{4}$, or $6\frac{3}{8}$ by $7\frac{7}{8}$, etc.

RULE.—*Add to the multiplicand the fraction of the multiplier and multiply this sum by the whole number; to the product add the product of the fractions.*

18. Multiply $6\frac{1}{4}$ by $5\frac{3}{4}$. Ans. $35\frac{3}{16}$.

The sum of $6\frac{1}{4}$ and $\frac{3}{4}$ is 7, so we simply say 5 times 7 are 35; to this we add the product of the fractions, $\frac{3}{4}$ times $\frac{1}{4}$ are $\frac{3}{16}=35\frac{3}{16}$. Ans.

19. Multiply $9\frac{1}{4}$ by $8\frac{2}{4}$. Ans. $78\frac{2}{16}$.

The sum of $9\frac{1}{4}$ and $\frac{2}{4}$ is $9\frac{3}{4}$, and 8 times $9\frac{3}{4}$ are 78, to which add the product of the fractions.

WHERE THE SUM OF THE FRACTIONS IS ONE.

To multiply any two numbers whose difference is *one* and the sum of the fractions is one.

RULE.—*Multiply the larger number, increased by one, by the smaller number; then square the fraction of the larger number, and subtract its square from one.*

PRACTICAL EXAMPLES FOR BUSINESS MEN.

1. What will $9\frac{1}{4}$ lbs. sugar cost at $8\frac{3}{4}$ cts. per lb.?

Here we multiply 9, increased by 1, by 8, thus: 8×10 are 80, and set down the result; then from 1 we subtract the square of $\frac{1}{4}$, thus: $\frac{1}{4}$ squared is $\frac{1}{16}$, and 1 less $\frac{1}{16}$ is $\frac{15}{16}$.

$$\begin{array}{r} 9\frac{1}{4} \\ 8\frac{3}{4} \\ \hline 80\frac{15}{16} \end{array}$$

2. What will $8\frac{2}{3}$ bu. coal cost at $7\frac{1}{3}$ cts. a bu.?

Here we multiply 8, increased by 1, by 7, thus: 7 times 9 are 63, and set down the result; then from 1 we subtract the square of $\frac{2}{3}$, thus: $\frac{2}{3}$ squared is $\frac{4}{9}$, and 1, less $\frac{4}{9}$, is $\frac{5}{9}$.

$$\begin{array}{r} 8\frac{2}{3} \\ 7\frac{1}{3} \\ \hline 63\frac{5}{9} \end{array}$$

3. What will $11\frac{2}{13}$ bu. seed cost at $\$10\frac{11}{13}$ a bu.?

Here we multiply 11, increased by 1, by 10 thus: 10 times 12 are 120, and set down the result; then from 1 we subtract the square of $\frac{2}{13}$, thus: $\frac{2}{13}$ squared is $\frac{4}{169}$, and 1 less $\frac{4}{169}$ is $\frac{165}{169}$.

$$\begin{array}{r} 11\frac{2}{13} \\ 10\frac{11}{13} \\ \hline 120\frac{165}{169} \end{array}$$

4. How many square inches in a floor $99\frac{3}{8}$ in. wide and $98\frac{5}{8}$ in. long? Ans. $9800\frac{55}{64}$.

Method of Operation.

Example First.

Multiply $6\frac{1}{4}$ by $6\frac{1}{4}$ in a single line.

Here we add $6\frac{1}{4}+\frac{1}{4}$, which gives $6\frac{1}{2}$; this multiplied by the 6 in the multiplier, $6\times6\frac{1}{2}$, gives 39, to which we add the product of the fractions; thus $\frac{1}{4}\times\frac{1}{4}$ gives $\frac{1}{16}$, added to 39 completes the product.

$$\begin{array}{r} 6\frac{1}{4} \\ 6\frac{1}{4} \\ \hline 39\frac{1}{16} \end{array}$$

Example Second.

Multiply $11\frac{1}{4}$ by $11\frac{3}{4}$ in a single line.

Here we would add $11\frac{1}{4}+\frac{3}{4}$, which gives 12; this multiplied by the 11 in the multiplier gives 132, to which we add the product of the fractions; thus $\frac{3}{4}\times\frac{1}{4}$ gives $\frac{3}{16}$, which added to 132 completes the product.

$$\begin{array}{r} 11\frac{1}{4} \\ 11\frac{3}{4} \\ \hline 132\frac{3}{16} \end{array}$$

Example Third.

Multiply $12\frac{1}{2}$ by $12\frac{3}{4}$ in a single line.

Here we add $12\frac{1}{2}+\frac{3}{4}$, which gives $13\frac{1}{4}$; this multiplied by the 12 in the multiplier, $12\times13\frac{1}{4}$, gives 159, to which add the product of the fractions; thus $\frac{3}{4}\times\frac{1}{2}$ gives $\frac{3}{8}$, which added to 159 completes the product.

$$\begin{array}{r} 12\frac{1}{2} \\ 12\frac{3}{4} \\ \hline 159\frac{3}{8} \end{array}$$

WHERE THE SUM OF THE FRACTIONS IS ONE.

To multiply any two *like* numbers together when the sum of the fractions is *one.*

Rule.—*Multiply the whole number by the next higher whole number, after which add the product of the fractions.*

N. B.—In the following examples the product of the fractions are obtained *first*, for convenience:

Practical Examples for Business Men.

Multiply $3\frac{3}{4}$ by $3\frac{1}{4}$ in a single line.

Here we multiply $\frac{1}{4}\times\frac{3}{4}$, which gives $\frac{3}{16}$, and set down the result; then we multiply the 3 in the multiplicand, increased by unity, by the 3 in the multiplier, 3×4, which gives 12 and completes the product.

$$\begin{array}{r} 3\frac{3}{4} \\ 3\frac{1}{4} \\ \hline 12\frac{3}{16} \end{array}$$

Multiply $7\frac{2}{5}$ by $7\frac{3}{5}$ in a single line.

Here we multiply $\frac{3}{5}\times\frac{2}{5}$, which gives $\frac{6}{25}$, and set down the result ; then we multiply the 7 in the multiplicand, increased by unity, by the 7 in the multiplier, 7×8, which gives 56, and completes the product.

$$\begin{array}{r} 7\frac{2}{5} \\ 7\frac{3}{5} \\ \hline 56\frac{6}{25} \end{array}$$

Multiply $11\frac{1}{3}$ by $11\frac{2}{3}$ in a single line.

Here we multiply $\frac{2}{3}\times\frac{1}{3}$, which gives $\frac{2}{9}$, and set down the result ; then we multiply the 11 in the multiplicand, increased by unity, by the 11 in the multiplier, 11×12, which gives 132, and completes the product.

$$\begin{array}{r} 11\frac{1}{3} \\ 11\frac{2}{3} \\ \hline 132\frac{2}{9} \end{array}$$

Example Fourth.

Multiply $16\frac{2}{3}$ by $16\frac{1}{3}$ in a single line.

Here we multiply $\frac{1}{3}\times\frac{2}{3}$ which gives $\frac{2}{9}$, and set down the result ; then we multiply the 16 in the multiplicand, increased by unity, by the 16 in the multiplier, 16×17, which gives 272, and completes the product.

$$\begin{array}{r} 16\frac{2}{3} \\ 16\frac{1}{3} \\ \hline 272\frac{2}{9} \end{array}$$

Example Fifth.

Multiply $29\frac{1}{2}$ by $29\frac{1}{2}$ in a single line.

Here we multiply $\frac{1}{2}\times\frac{1}{2}$ which gives $\frac{1}{4}$, and set down the result ; then we multiply the 29 in the multiplicand, increased by unity, by the 29 in the multiplier, 29×30, which gives 870, and completes the product.

$$\begin{array}{r} 29\frac{1}{2} \\ 29\frac{1}{2} \\ \hline 870\frac{1}{4} \end{array}$$

Example Sixth.

Multiply $999\frac{3}{8}$ by $999\frac{5}{8}$ in a single line.

Here we multiply $\frac{5}{8}\times\frac{3}{8}$, which gives $\frac{15}{64}$, and set down the result; then we multiply the 999 in the multiplicand, increased by unity, by the 999 in the multiplier, 999×1000, which gives 999000, and completes the product.

$$\begin{array}{r} 999\frac{3}{8} \\ 999\frac{5}{8} \\ \hline 999000\frac{15}{64} \end{array}$$

NOTE.—The system of multiplication introduced in the preceding examples applies to all numbers. Where the sum of the fractions is *one*, and the whole numbers are alike, or differ by *one*, the learner is requested to study well these useful properties of numbers.

WHERE THE FRACTIONS HAVE A LIKE DENOMINATOR.

To multiply any two *like* numbers together, each of which has a fraction with a *like* denominator, as $4\frac{3}{8}\times4\frac{7}{8}$, or $11\frac{1}{4}\times11\frac{3}{4}$, or $10\frac{2}{5}\times10\frac{1}{5}$, etc.

RULE.—*Add to the multiplicand the fraction of the multiplier, and multiply this sum by the whole number, after which add the product of the fractions.*

Practical Examples for Business Men.

N. B.—In the following example the sum of the fractions is *one:*

1. What will $9\frac{3}{4}$ lbs. beef cost at $9\frac{1}{4}$ cts. a lb.?

The sum of $9\frac{3}{4}$ and $\frac{1}{4}$ is 10, so we simply say 9 times 10 are 90; then we add the product of the fractions, $\frac{1}{4}$ times $\frac{3}{4}$ are $\frac{3}{16}$.

$$\begin{array}{r} 9\frac{3}{4} \\ 9\frac{1}{4} \\ \hline 90\frac{3}{16} \end{array}$$

N. B.—In the following example the sum of the fractions is *less* than *one:*

1. What will $8\frac{1}{4}$ yds. tape cost at $8\frac{2}{4}$ cts. a yd.?

The sum of $8\frac{1}{4}$ and $\frac{2}{4}$ is $8\frac{3}{4}$, so we simply say 8 times $8\frac{3}{4}$ are 70; then we add the product of the fractions, $\frac{2}{4}$ times $\frac{1}{4}$ are $\frac{2}{16}$, or $\frac{1}{8}$.

$$\begin{array}{r} 8\frac{1}{4} \\ 8\frac{2}{4} \\ \hline 70\frac{1}{8} \end{array}$$

N. B.—In the following example the sum of the fractions is *greater* than *one:*

3. What will $4\frac{3}{8}$ yds. cloth cost at $\$\frac{7}{8}$ a yd.?

The sum of $4\frac{3}{8}$ and $\frac{7}{8}$ is $5\frac{1}{4}$, so we simply say 4 times $5\frac{1}{4}$ are 21; then we add the product of the fractions, $\frac{7}{8}$ times $\frac{3}{8}$ are $\frac{21}{64}$.

$$\begin{array}{r} 4\frac{3}{8} \\ 4\frac{7}{8} \\ \hline 21\frac{21}{64} \end{array}$$

N. B.—Where the fractions have different denominators reduce them to a common denominator.

RAPID PROCESS FOR MULTIPLYING MIXED MUMBERS.

A valuable and useful rule for the accountant in the practical calculations of the counting room.

To multiply any two numbers together, each of which involves the fraction $\frac{1}{2}$—as $7\frac{1}{2} \times 9$, etc.

RULE.—*To the product of the whole numbers add half their sum, plus* $\frac{1}{4}$.

Examples for Mental Operations.

1. What will $3\frac{1}{2}$ doz. eggs cost at $7\frac{1}{2}$ cts. a doz.?

Here the sum of 7 and 3 is 10, and half this sum is 5, so we simply say 7 times 3 are 21 and 5 are 26, to which we add $\frac{1}{4}$.

$$\begin{array}{r} 3\frac{1}{2} \\ 7\frac{1}{2} \\ \hline 26\frac{1}{4} \end{array}$$

N. B.—If the sum be an odd number call it one less, to make it even, and in such cases the fraction must be $\frac{3}{4}$.

2. What will $11\frac{1}{2}$ lbs. cheese cost at $9\frac{1}{2}$ cts. a lb.?

3. What will $8\frac{1}{2}$ yds. tape cost at $15\frac{1}{2}$ cts. a yd.?

4. What will $7\frac{1}{2}$ lbs. rice cost at $13\frac{1}{2}$ cts. a lb.?

5. What will $10\frac{1}{2}$ bu. coal cost at $12\frac{1}{2}$ cts. a bu.?

REASON.—In explaining the above rule we add half their sum, because half of either number added to half the other would be half their sum, and we add $\frac{1}{4}$ because $\frac{1}{2}\times\frac{1}{2}$ is $\frac{1}{4}$. The same principle will multiply any two numbers together, each of which has the same fraction—for instance, if the fraction was $\frac{1}{3}$ we would add one third their sum; if $\frac{3}{4}$, we would add three fourths their sum, etc.; and then, to complete the multiplication, we would add, of course, the product of the fractions.

6. Multiply $4\frac{3}{8}$ by $4\frac{7}{8}$. Ans. $21\frac{21}{64}$.

The sum of $4\frac{3}{8}$ and $\frac{7}{8}$ is $5\frac{1}{4}$, and 4 times $5\frac{1}{4}$ is 21; add $\frac{3}{8}\times\frac{7}{8}=\frac{21}{64}$. $21\frac{21}{64}$ Ans.

To multiply any two numbers together, each of which involves the fraction $\frac{1}{2}$.

RULE.—*To the product of the whole numbers add half their sum, plus $\frac{1}{4}$.*

7. Multiply $3\frac{1}{2}\times7\frac{1}{2}$. Ans. $26\frac{1}{4}$.

Solution.—The sum of 3 and 7 are 10, and one half this sum is 5, so we say 7 times 3 are 21 and 5 are 26, to which we annex $\frac{1}{4}$. $26\frac{1}{4}$ Ans.

8. What will $7\frac{1}{2}$ lbs. cheese cost at $13\frac{1}{2}$ cts. a lb.? Ans. $1.01\frac{1}{4}$.

REMARK.—If the sum be an odd number call it one less, to make it even; in which case the fraction must be $\frac{3}{4}$.

9. What will $8\frac{1}{2}$ lbs. of sugar cost at $15\frac{1}{2}$ cts. a lb.? Ans. 1.31\frac{3}{4}$.

Here, 8+15=23, being an odd number, we make it one less, 22, one half of which is 11. Then, 8 times 15 are 120, and 11 are 131, to which we add $\frac{3}{4}$.

The same principle will multiply any two numbers together, each of which has the same fraction. For instance, if the fraction was $\frac{1}{5}$, we would add one fifth their sum; if $\frac{3}{4}$, we would add three fourths their sum; if $\frac{2}{3}$, add two thirds their sum, etc., after which, of course, add the product of their fractions.

10. Multiply $8\frac{2}{3}\times7\frac{2}{3}$. Ans. $66\frac{4}{9}$.

The sum of 8 and 7 are 15, two thirds of which is 10. We then say, 8 times 7 are 56 and 10 make 66, and add $\frac{2}{3}\times\frac{2}{3}=\frac{4}{9}$.

Business men generally, in multiplying, only care about having the answer correct to the nearest cent —that is, they disregard the fraction. When it is a half cent or more they call it another cent; if less than half a cent, they drop it. Therefore, to multiply any two numbers to the nearest unit we give the following

General Rule.

I. *Multiply the whole number in the multiplicand by the fraction in the multiplier to the nearest unit.*

II. *Multiply the whole number in the multiplier by the fraction in the multiplicand to the nearest unit.*

III. *Multiply the whole numbers together, and add the three products in your mind as you proceed.*

REMARK.—This rule is very simple and true, and there being no such thing as a fraction to add in, there is scarcely any liability to error or mistake.

The work can generally be done mentally, for only easy fractions *actually occur in business.*

11. Multiply $9\frac{1}{3}$ by $8\frac{1}{4}$. Ans. 77.

Solution.—$\frac{1}{4}$ of 9 is nearer 2 than 3, and $\frac{1}{3}$ of 8 is nearer 3 than 2. Make the nearest whole number the quotient. 2 and 3 are 5, so we simply say 8 times 9 are 72 and 5 are 77. Ans.

12. Multiply $11\frac{2}{3}$ by $7\frac{4}{5}$. Ans. 91.

Here, $\frac{4}{5}$ of 11 to the nearest unit is 9, and $\frac{2}{3}$ of 7 to the nearest unit is 5; then $9+5=14$, so we say 7 times 11 are 77, and 14 are 91. Ans.

The component factors of a number are such factors as multiplied together will produce that number—thus, 3 and 4 are compound factors of 12, because $3\times4=12$; also, 2, 2 and 3, because $2\times2\times3=12$; also, $6\times2=12$.

13. Multiply $128\frac{2}{3}$ by 25, by business method.

Here $\frac{2}{3}$ of 25 to the nearest unit is 17, so we simply say, 25 times 128 are 3200, and 17 are 3217, the answer.

$$\begin{array}{r} 128\frac{2}{3} \\ 25 \\ \hline 3217 \end{array}$$

Practical Examples for Business Men.

1. What is the cost of $17\frac{1}{2}$ lbs. sugar at $18\frac{3}{4}$ cts. per lb.?

Here $\frac{3}{4}$ of 17 to the nearest unit is 13, and $\frac{1}{2}$ of 18 is 9, 13 plus 9 is 22, so we simply say 18 times 17 are 306, and 22 are 328, the answer.

$$\begin{array}{r} 17\frac{1}{2} \\ 18\frac{3}{4} \\ \hline \$3.28 \end{array}$$

2. What is the cost of 11 lbs. 5 oz. of butter at $33\frac{1}{3}$ cts. per lb.?

Here $\frac{1}{3}$ of **11** to the nearest unit is 4, and $\frac{5}{16}$ of 33 to the nearest unit is 10, then 4 +10 is 14, so we simply say 33 times 11 are 363, and 14 are 377, the answer.

$$\begin{array}{r} 11\frac{5}{16} \\ 33\frac{1}{3} \\ \hline \$3.77 \end{array}$$

3. What is the cost of 17 doz. and 9 eggs, at $12\frac{1}{2}$ cts. per doz.?

Here $\frac{1}{2}$ of 17 to the nearest unit is 9, and $\frac{9}{12}$ of 12 is 9; then 9+9=18, so we simply say 12 times 17 are 204, and 18 are 222, the answer.

$$\begin{array}{r} 17\frac{9}{12} \\ 12\frac{1}{2} \\ \hline \$2.22 \end{array}$$

4. What will be the cost of $15\frac{3}{4}$ yds. calico at $12\frac{1}{2}$ cts. per yd.? Ans. $1.97.

N. B.—To multiply by aliquot parts of 100 see page 44.

Rapid Process of Marking Goods.

A VALUABLE HINT TO MERCHANTS AND ALL RETAIL DEALERS IN FOREIGN AND DOMESTIC DRY GOODS.

Retail merchants, in buying goods by wholesale, buy a great many articles by the dozen, such as boots and shoes, hats and caps, and notions of various kinds. Now the merchant, in buying, for instance, a dozen hats, knows exactly what one of those hats will retail for in the market where he deals; and, unless he is a good accountant, it will often take him some time to determine whether he can afford to purchase the dozen hats and make a living profit in selling them by the single hat; and, in buying his goods by auction, as the merchant often does, he has not time to make the calculation before the goods are cried off; he therefore loses the chance of making good bargains by being afraid to bid at random, or if he bids, and the goods are cried off, he may have made a poor bargain by bidding thus at a venture. It then becomes a useful and practical problem to determine *instantly* what per cent. he would gain if he retailed the hats at a certain price.

To tell what an article should retail for to make a profit of 20 per cent. :

Rule.—*Divide what the articles cost per dozen by* 10, *which is done by removing the decimal point one place to the left.*

For instance, if hats cost \$17.50 per dozen, remove the decimal point one place to the left, making \$1.75, what they should be sold for apiece to gain 20 per cent. on the cost. If they cost \$31.00 per dozen they should be sold at \$3.10 apiece, etc. We take 20 per cent. as the basis for the following reasons, viz., because we can determine instantly, by simply removing the decimal point without changing a figure ; and, if the goods would not bring at least 20 per cent. profit in the home market, the merchant could not afford to purchase, and would look for goods at lower figures.

Reason.—The reason for the above rule is obvious —for if we divide the cost of a dozen by 12 we have the cost of a single article ; then, if we wish to make 20 per cent. on the cost (cost being $\frac{1}{1}$ or $\frac{5}{5}$), we add the 20 per cent., which is $\frac{1}{5}$, to the $\frac{5}{5}$, making $\frac{6}{5}$ or $\frac{12}{10}$; then, as we multiply the cost, divided by 12, by the $\frac{12}{10}$ to find at what price one must be sold to gain 20 per cent., it is evident that the 12s will cancel, and leave the cost of a dozen to be divided by 10, which is done by removing the decimal point one place to the left.

1. If I buy 2 doz. caps, at \$7.50 per doz., what shall I retail them at to make 20%? Ans. 75 cts.

2. When a merchant retails a vest at \$4.50 and makes 20% what did he pay per doz.? Ans. \$45.

3. At what price should I retail a pair of boots, that cost \$85 per doz., to make 20%? Ans. \$8.50.

Rapid Process of Marking Goods at Different per cents.

Now, as removing one decimal point one place to the left, on the cost of a dozen articles, gives the selling price of a single one with 20 per cent. added to the cost, and as the cost of any article is 100 per cent., it is obvious that the selling price would be 20 per cent. more, or 120 per cent.; hence, to find 50 per cent. profit, which would make the selling price 150 per cent., we would first find 120 per cent., then add 30 per cent. by increasing it one fourth itself. To make 40 per cent., add 20 per cent., by increasing it one sixth itself; for 35 per cent. increase it one eighth itself, etc. Hence, to mark an article at any per cent. profit we have the following

General Rule.

First find 20 *per cent. profit by removing the decimal point one place to the left on the price the articles cost a dozen; then, as* 20 *per cent. profit is* 120 *per cent., add to or subtract from this amount the fractional part that the required per cent. added to* 100 *is more or less than* 120.

Merchants, in marking goods, generally take a per cent. that is an aliquot part of 100, as 25%, 33⅓%, 50%, etc. The reason they do this is because it makes it much easier to add such a per cent. to the cost. For instance, a merchant could mark almost a dozen articles at 50 per cent. profit in the time it would take him to mark a single one at 49 per cent. For the benefit of the student, and for the convenience of business men in marking goods, we have arranged the following

Table

FOR MARKING ALL ARTICLES BOUGHT BY THE DOZEN.

N. B.—Most of these are used in business.

To	make	20%	remove	the	point	one place to the left			
"	"	80%	"	"	"	and	add	$\frac{1}{2}$	itself.
"	"	60%	"	"	"	"	"	$\frac{1}{3}$	"
"	"	50%	"	"	"	"	"	$\frac{1}{4}$	"
"	"	44%	"	"	"	"	"	$\frac{1}{5}$	"
"	"	40%	"	"	"	"	"	$\frac{1}{6}$	"
"	"	$37\frac{1}{2}\%$	"	"	"	"	"	$\frac{1}{7}$	"
"	"	35%	"	"	"	"	"	$\frac{1}{8}$	"
"	"	$33\frac{1}{3}\%$	"	"	"	"	"	$\frac{1}{9}$	"
"	"	32%	"	"	"	"	"	$\frac{1}{10}$	"
"	"	30%	"	"	"	"	"	$\frac{1}{12}$	"
"	"	28%	"	"	"	"	"	$\frac{1}{15}$	"
"	"	26%	"	"	"	"	"	$\frac{1}{20}$	"
"	"	25%	"	"	"	"	"	$\frac{1}{24}$	"
"	"	$12\frac{1}{2}\%$	"	"	"	subtract		$\frac{1}{16}$	"
"	"	$16\frac{2}{3}\%$	"	"	"	"		$\frac{1}{36}$	"
"	"	$18\frac{3}{4}\%$	"	"	"	"		$\frac{1}{96}$	"

If I buy 1 doz. shirts for $28.00 what shall I retail them for to make 50%? Ans. $3 50.

EXPLANATION.—Remove the point one place to the left and add on $\frac{1}{4}$ itself.

WHERE THE MULTIPLIER IS AN ALIQUOT PART OF 100.

Merchants, in selling goods, generally make the price of an article some aliquot part of 100, as in selling sugar at $12\frac{1}{2}$ cents a pound, or 8 pounds for 1 dollar, or in selling calico for $16\frac{2}{3}$ cents a yard, or 6 yards for 1 dollar, etc. And to become familiar with all the aliquot parts of 100, so that you can apply them readily when occasion requires, is, perhaps, the most useful, and, at the same time, one of the easiest arrived at of all the computations the accountant must perform in the practical calculations of the counting-room.

Table of the Aliquot parts of 100 and 1,000.

N. B.—Most of these are used in business.

$12\frac{1}{2}$ is $\frac{1}{8}$ part of 100.	$8\frac{1}{3}$ is $\frac{1}{12}$ part of 100.
25 is $\frac{2}{8}$ or $\frac{1}{4}$ of 100.	$16\frac{2}{3}$ is $\frac{2}{12}$ or $\frac{1}{6}$ of 100.
$37\frac{1}{2}$ is $\frac{3}{8}$ part of 100.	$33\frac{1}{3}$ is $\frac{4}{12}$ or $\frac{1}{3}$ of 100.
50 is $\frac{4}{8}$ or $\frac{1}{2}$ of 100.	$66\frac{2}{3}$ is $\frac{8}{12}$ or $\frac{2}{3}$ of 100.
$62\frac{1}{2}$ is $\frac{5}{8}$ part of 100.	$83\frac{1}{3}$ is $\frac{10}{12}$ or $\frac{5}{6}$ of 100.
75 is $\frac{6}{8}$ or $\frac{3}{4}$ of 100.	125 is $\frac{1}{8}$ part of 1,000.
$87\frac{1}{2}$ is $\frac{7}{8}$ part of 100.	250 is $\frac{2}{8}$ or $\frac{1}{4}$ of 1,000.
$6\frac{1}{4}$ is $\frac{1}{16}$ part of 100.	375 is $\frac{3}{8}$ part of 1,000.
$18\frac{3}{4}$ is $\frac{3}{16}$ part of 100.	625 is $\frac{5}{8}$ part of 1,000.
$31\frac{1}{4}$ is $\frac{5}{16}$ part of 100.	875 is $\frac{7}{8}$ part of 1,000.

To multiply by an aliquot part of 100.

RULE.—*Add two ciphers to the multiplicand, then take such part of it as the multiplier is part of* 100.

N. B.—If the multiplicand is a mixed number reduce the fraction to a decimal of two places before dividing.

To multiply by an aliquot part of 100.

RULE.—*Add two ciphers to the multiplicand, then take such part of it as the multiplier is part of* 100.

To multiply by $12\frac{1}{2}$, add two ciphers and divide by 8.

1. Multiply 2592 by $12\frac{1}{2}$. Product, 32400.

$$
\begin{array}{r}
8)\underline{259200} \\
32400
\end{array}
$$

To multiply by $37\frac{1}{2}$, annex two ciphers and take $\frac{3}{8}$ of it.

2. Multiply 1432 by $37\frac{1}{2}$. Product, 53700.

$$
\begin{array}{r}
8)\underline{14300} \\
17900 \\
\underline{3} \\
53700
\end{array}
$$

To multiply by $6\frac{2}{3}$, add two ciphers and divide by 15; or add one cipher and multiply by $\frac{2}{3}$.

3. Multiply 6525 by $6\frac{2}{3}$. Product, 43500.

$$
\begin{array}{r}
15)\underline{652500} \\
43500
\end{array}
\quad \text{or,} \quad
\begin{array}{r}
3)\underline{65250} \\
21750 \\
\underline{2} \\
43500
\end{array}
$$

To multiply by $87\frac{1}{2}$, add two ciphers, divide by 8 and subtract the quotient, or multiply the quotient by 7.

4. Multiply 6768 by $87\frac{1}{2}$. Product, 592200.

8)676800	or,	8)676800
84600		84600
592200		7
		592200

To multiply by 75, add two ciphers and subtract $\frac{1}{4}$.

5. Multiply 4968 by 75. Ans. 372600.

$$\begin{array}{r} 4)496800 \\ 124200 \\ \hline 372600 \end{array}$$

To multiply by 125, add three ciphers and divide by 8.

6. Multiply 3467 by 125. Ans. 433375.

$$\begin{array}{r} 8)3467000 \\ \hline 433375 \end{array}$$

To multiply by 875, add three ciphers and subtract $\frac{1}{8}$.

7. Multiply 25136 by 875. Ans. 21994000.

$$\begin{array}{r} 8)25136000 \\ 3142000 \\ \hline 21994000 \end{array}$$

Every fact of this kind, though extremely simple, will be found very useful many times, and should be known by all who seek to be skilful in figures.

THE GREAT SECRET OF MATHEMATICS REVEALED.

Nearly every problem in mathematics, of whatever name, will come under one of the three following heads, and can be resolved by the appropriate rule belonging to them. For this reason they should be well *committed to memory* and *carefully studied* until the learner becomes *perfectly familiar* with *each* and *all* of them. We will first give them all, after which we will exemplify each one of them separately:

1. The price of ONE and the QUANTITY being given, to find the COST of the quantity.

RULE.—*Multiply the price of* ONE *by the* QUANTITY.

2. The COST and the QUANTITY being given, to find the price of ONE.

RULE.—*Divide the* COST *by the* QUANTITY.

3. The price of ONE and the COST of a QUANTITY being given, to find the QUANTITY.

RULE.—*Divide the* COST *of the* QUANTITY *by the price of* ONE.

CASE I.—The price of one and the quantity being given, to find the cost of the quantity.

EXAMPLE 1.—If one acre of land cost $15, what will 60 acres cost?

Multiply the price of one acre, $15, by the quantity, 60 acres: 15×60=$900. Ans.

2. What commission must be paid for collecting \$17380 at $3\frac{1}{2}$ per cent.?

Multiply the price of collecting one dollar ($3\frac{1}{2}$ cents) by the quantity, \$17380: $.03\frac{1}{2} \times \$17380 =$ \$608.30. Ans.

3. A broker negotiates a bill of exchange of \$2890 for $\frac{4}{5}$ per cent. commission. How much is his brokerage?

Multiply the price of negotiating one dollar ($\frac{4}{5}$ of a cent) by the quantity, \$2890. \$23.12. Ans.

4. If the stock of an insurance company sells at 5 per cent. below par, what will \$1200 of the stock cost?

If the stock was at par one dollar's worth of stock would be worth \$1, but as it is 5 per cent. below par, one dollar of stock is only worth 95 cents; therefore, multiply 95 cents, the price of one, by the quantity, \$1200. \$1140. Ans.

5. What will cost \$5364 stock in the Bank of Orleans, at 9 per cent. above par?

Since it is above par one dollar of stock is worth \$1.09; therefore, multiply \$1.09 by \$5364. \$5846.76. Ans.

6. What is the interest on \$512 for three years, at 7 per cent.?

The price of one dollar for three years at 7 per cent. is 21 cents; therefore, multiply the price, 21 cents, by \$512. \$107.52. Ans.

7. What premium must be paid for \$4572.80 insurance at $2\frac{1}{2}$ per cent.?

Multiply $2\frac{1}{2}$ cents, the price of one dollar, by the quantity, \$4572.80. \$114.32. Ans.

CASE II.—The cost and the quantity being given, to find the price of one.

EXAMPLE 1.—If 25 acres of land cost \$175, what will one acre cost?

Divide the cost of the quantity, \$175, by the quantity, 25. \$7. Ans.

2. A man having \$125 lost \$5. What per cent. of his money did he lose?

Divide the cost of the quantity, \$5, by the quantity, \$125: \$5÷\$125=.04 per cent. Ans.

3. I lent \$450 for one year, and received for interest \$31.50. What was the rate per cent.?

Divide the cost of the quantity, \$31.50, by the quantity, \$450: \$31.50÷\$450=.07 per cent. Ans.

CASE III.—The price of one and the cost of a quantity being given, to fine the quantity.

EXAMPLE 1.—At \$6 a barrel for flour how many barrels can be bought for \$840?

Divide the cost of the quantity by the price of one: \$840÷\$6=140. Ans.

2. A man lost \$5, which was 4 per cent. of all the money he had. How much had he at first?

Divide the cost of the quantity, \$5, by the price of one, .04: \$5÷.04=\$125.

3. What amount of stock can be bought for \$9682, allowing 3 per cent. brokerage?

Every dollar's worth of stock costs $1.03 ; therefore divide the cost of the quantity, $9682, by the price of one : $9682÷$1.03=$9400. Ans.

4. What principal, in 2 years 6 months, at 7 per cent., will amount to $88.125 ?

The price of $1 for 2 years 6 months will be $1.175; therefore divide the cost of the quantity, $88.125, by $1.175. $88.125÷$1.175=$75. Ans.

5. In what time will $360 gain $86.40 interest at 6 per cent. ?

The price of $360 for one year at 6 per cent. will be $21.60 ; therefore, divide the cost, $86.40, by the price of one year, which will give the number of years. $86.40÷$21.60=4 years. Ans.

INTEREST

is a sum paid for the use of money.

Principal is the sum for the use of which interest is paid.

Amount is the sum of the principal and interest.

Rate per cent., commonly expressed decimally as hundredths, is the sum per cent. paid for the use of one dollar annually.

Simple Interest is the sum paid for the use of the principal only during the whole time of the loan.

Legal Interest is the rate per cent. established by law.

Usury is illegal interest, or a greater per cent than the legal rate.

It is contended by many statesmen that the rate of interest should not be established by statute, but that money is only a commodity that, like every other article of traffic, should be governed by the law of supply and demand. If money is scarce the rate would be high ; if plenty, then low. But as banks and other great monied institutions have the power, to a great extent, of controlling the quantity of money in the market, thereby oppressing the great majority of the people, and taking advantage of the times of scarcity, public opinion, at least, has established the law of *usury.*

The Jews appear to be about the first nation among whom we find a distinct class called "money changers" or "lenders," and among them we find a law existed that they should not take interest of their brethren, though they were permitted to take it of foreigners. "Thou shalt not lend upon usury to thy brother—usury of money, usury of victuals, usury of anything that is lent upon usury ; unto a stranger thou mayest lend upon usury ; but unto thy brother thou shalt not lend upon usury."—(*Deut.* xxiii, 19, 20.) After the dispersion of the Jews they wandered through the earth—but they yet remain a distinct people, mixing but not becoming assimilated with the people among whom they reside. Still looking forward to the period when they shall return to the promised land, they seldom engage in permanent business, but pursue traffic, especially dealing in money ; and if their national policy forbids their taking interest of each other, they show no backwardness in taking it unsparingly of the rest of mankind. For ages they have been the money lenders of Europe, and we may safely attribute to this circumstance the prejudice, in some measure, that still exists, even in our own country, against such as pursue this business as a profession. The prejudice of the Christian against the Jew has been transferred to his occupation, and from the days of the inexorable Shylock, contending for his pound of flesh, down to the present time, the grasping money lender, no less than the grinding dealer in other matters, has been sneeringly called a Jew.

The rate of legal interest varies in different States, and we subjoin a table giving the legal rate as well as that allowed by contract. When the rate per cent. is not specified in accounts, notes, mortgages, contracts, etc., the legal rate is always understood.

Rates of Interest and Statute Limitations in the United States.

States.	Legal Rate.	Allowed by Contract.	Penalty for Usury.	Stat. Lim. Open Acct.	Stat. Lim. Notes.	Stat. Lim. Judgments.
				Yrs.	Yrs.	Yrs.
Alabama....	8	..	Forfeiture of entire interest........	3	6	20
Arkansas....	6	10	Usurious contracts void............	3	5	10
California...	10	18	....	1	4	5
Connecticut.	6	..	Forfeiture of entire interest........	6	6	17
Delaware...	6	..	Forfeiture of entire principal......	3	6	..
Florida......	6	8	Forfeiture of entire interest........	5	5	..
Georgia.....	7	..	Forfeiture of excess of interest.....	4	6	20
Illinois......	6	10	Forfeiture of entire interest........	5	5	20
Indiana.....	6	..	Usurious interest recoverable......	6	20	20
Iowa........	6	10	Usurious interest recoverable......	..	5	20
Kentucky...	6	..	Usurious excess void..............	1	5	15
Louisiana...	5	8	Forfeiture of entire interest........	3	5	..
Maine.......	6	..	Usurious excess void..............	6	6	20
Maryland...	6	..	Forfeit of usury..................	3	3	12
Mass........	6	..	Forfeit threefold usurious int. taken	6	6	..
Michigan....	7	10	Usurious excess void..............	6	6	..
Minnesota...	7	free	....	..	6	10
Mississippi..	6	10	Forfeiture of interest...............	3	6	7
Missouri....	6	10	Forfeiture of interest...............	5	10	20
N. Hampsh'e	6	..	Forfeit threefold usurious int. taken	6	6	20
New Jersey.	6	..	Contract void.....................	6	16	16
New York..	7	..	Contract void. Fine not over $100, impris'mt not over 6 mos., or both	6	6	20
N. Carolina..	6	..	Forfeit double the debt.	3	3	..
Ohio........	6	..	Usurious excess void..............	6	15	20
Pennsylvania	6	..	Forfeit entire principal and interest	6	6	20
Rhode Island	6	..	Usurious excess void..............	6	6	..
S. Carolina..	7	..	Forfeit entire interest..............	4	4	..
Tennessee...	6	10	Fine at least $10..................	3	6	16
Texas.......	8	12	Forfeit entire interest..............	2	4	..
Vermont....	6	..	Usurious excess void..............	6	6	8
Virginia.....	6	..	Contract void.....................	5	5	20
Wisconsin...	7	12	Forfeit entire debt................	6	6	..

To find the interest if the time consists of years.

Rule.—*Multiply the principal by the rate per cent., and that product by the number of years.*

EXAMPLE 1.—What is the interest of $150 for 3 years, at 8 per cent.?

```
  $150
   .08
  ----
 12.00
     3
 -----
$36.00 Ans.
```

The decimal for 8 per cent. is .08. There being two places of decimals in the multiplier we point off two places in the product.

To find the interest when the time consists of years and months.

RULE.—*Reduce the time to months. Multiply the principal by the rate per cent., divide the product by* 12, *and the quotient multiplied by the number of months will be the interest required.*

OR, BY CANCELLATION.—*Place the principal, rate and time in months on the right of the line, and* 12 *on the left, then cancel.*

2. Find the interest of $240 for 2 years and 7 months, at 7 per cent.

```
Principal,        $240
Rate,              .07
                  ----
              12)16.80
                 -----
                  1.40
2 yrs+7 mos.        31
                  ----
                  1.40
                 4.20
                ------
                $43.40 Ans.
```

BY CANCELLATION.

$$\begin{array}{r|l} & \cancel{\$240}\ \ 20 \\ \cancel{12} & 7 \\ & 31 \end{array}$$

$20 \times 7 \times 31 = \$43.40$ Ans.

SIMPLE INTEREST BY CANCELLATION.

RULE.—*Place the principal, time and rate per cent. on the right hand side of the line. If the time consists of years and months, reduce them to months, and place* 12 (*the number of months in a year*) *on the left hand side of the line. Should the time consist of months and days, reduce them to days, or decimal parts of a month. If reduced to days, place* 36 *on the left. If to decimal parts of a month, place* 12 *only as before.*

Point off two decimal places when the time is in months, and three decimal places when the time is in days.

NOTE.—If the principal contains cents, point off four decimal places when the time is in months, and five decimal places when the time is in days.

(We place 36 on the left, because there are 360 interest days in a year. Custom has made this lawful.)

EXAMPLE 1.—What is the interest on $60 for 117 days, at 6 per cent.?

OPERATION.

Here 117×0		$\cancel{6}0$	Both 6s on the right
must be the	$3\cancel{6}$	$\cancel{6}$	cancel 36 on the
answer.		117	left, and we have
			nothing to divide
		$1.170 Ans.	by.

In this case we point off three decimal places, because the time is in days. If the time had been 117 months we would have pointed off but two decimal places.

EXAMPLE 2.—What is the interest of $96.50 for 90 days, at 6 per cent.?

OPERATION.

$$\begin{array}{r|l} & 96.50 \\ \cancel{6}—\cancel{36} & \cancel{90}—15 \\ & \cancel{6} \end{array} \qquad \begin{array}{r} 9650 \\ 15 \\ \hline 1.44.750 \text{ Ans.} \end{array}$$

Now cancel 6 in 36 and the quotient 6 into 90, and we have no divisor left. Hence, 15×96.50 must be the answer.

NOTE.—As there are cents in the principal we point off five decimals—three for days and two for cents. Pay no attention to the decimal point until the close of the operation.

EXAMPLE 3.—What is the interest of $480 for 361 days at 6 per cent.?

$$\begin{array}{r|l} & \cancel{480}—80 \\ \cancel{6}—\cancel{36} & 361 \\ & \cancel{6} \end{array} \qquad \begin{array}{r} 361 \\ 80 \\ \hline \$28.880 \text{ Ans.} \end{array}$$

Now cancel 6 in 36 and the quotient 6 into 480, and we have no divisor left. Hence, 80×361 must be the answer.

EXAMPLE 4.—What is the interest of $720 for 9 months, at 7 per cent.?

$$\begin{array}{r|l} & \cancel{720}—60 \\ \cancel{12} & 9 \\ & 7 \end{array} \qquad \begin{array}{r} 60 \\ 9 \\ \hline 540 \\ 7 \\ \hline \$37.80 \text{ Ans.} \end{array}$$

Now cancel 12 in 720 and there is nothing left to divide by. Hence, 60×9×7 must be the answer.

N. B.—When interest is required on any sum for days only, it is a universal custom to consider 30 days a month, and 12 months a year; and, as the unit of time is a year, the interest of any sum for *one day* is $\frac{1}{360}$ what it would be for a year. For 2 days, $\frac{2}{360}$, etc.; hence, if we multiply by the days, we must divide by 360, or divide by 36 and save labor. The old form of this method was to place 360, or 12 and 30, on the left of the line, but using 36 is much shorter.

WHEN THE DATES ARE NOT DIVISIBLE BY THREE.

NOTE.—When the time consists of months and days, and the days are not divisible by three, *reduce the time to days.*

EXAMPLE 5.—What is the interest of $960 for 11 months and 20 days at 6 per cent.?

		Months.	Days.
OPERATION.		11	20=350 days.
	$\cancel{96}\cancel{0}$—160		350
$\cancel{6}$—36	350		160
	6		$56.000

Now cancel 6 in 36 and the quotient 6 into 960, and we have no divisor left. Hence, 160×350 must be the answer.

EXAMPLE 6.—What is the interest of $173 for 8 months and 16 days, at 9 per cent?

		Months.	Days.
OPERATION.		8	16=256 days.
	173		173
$\cancel{4}$—$\cancel{36}$	$\cancel{9}$		64
	$\cancel{256}$—64		$11.072 Ans.

Now cancel 9 in 36 and the quotient 4 into 256, and we have no divisor left. Hence, 64×173 must be the answer.

N. B.—Let the pupil remember that this is a general and universal method, equally applicable to any per cent. or any required time, and all other rules must be reconcilable to it; and, in fact, all other rules are but modifications of this.

Bankers' Method of Computing Interest at 6 per cent. for any Number of Days.

Rule.—*Draw a perpendicular line, cutting off the two right hand figures of the* $, *and you have the interest for* 60 *days at* 6 *per cent.*

Note.—The figures on the left of the line are dollars, and those on the right are decimals of dollars.

Example 1.—What is the interest of $423, 60 days, at 6 per cent.?

$423=the principal.
$4 | 23 cts.=interest for 60 days.

Note.—When the time is more or less than 60 days first get the interest for 60 days, and from that to the time required.

Example 2.—What is the interest of $124 for 15 days, at 6 per cent.?

Days. Days.
15=¼ of 60

$124=principal.
4)1 | 24 cts.=interest for 60 days.

| 31 cts.=interest for 15 days.

EXAMPLE 3.—What is the interest of $123.40 for 90 days, at 6 per cent.?

Days. Days. Days.
90=60+30

$123.40=principal.
2)1 | 2340=interest for 60 days.
| 6170=interest for 30 days.

Ans. $1 | 851=interest for 90 days.

EXAMPLE 4.—What is the interest of $324 for 75 days, at 6 per cent.?

Days. Days. Days.
75=60+15

$324=principal.
4)3 | 24 cts. interest for 60 days.
| 81 cts. interest for 15 days

Ans. $4 | 05 cts. interest for 75 days.

REMARK.—This system of computing interest is very easy and simple, especially when the days are aliquot parts of 60, and one simple division will suffice. It is used extensively by a large majority of our most prominent bankers; and, indeed, is taught by most all commercial colleges as the shortest system of computing interest.

Method of Calculating at Different per cents.

This principle is not confined alone to 6 per cent., as many suppose who teach and use it. It is their custom *first* to find the interest at 6 per cent., and from that to other per cents; but it is equally applicable for *all* per cents., from 1 to 15, inclusive.

The following table shows the different per cents., with the time that a given number of $ will amount to the same number of cents when placed at interest:

RULE.—*Draw a perpendicular line, cutting off the two right hand figures of $, and you have the interest at the following per cents.:*

Interest at 4 per cent. for 90 days.
Interest at 5 per cent. for 72 days.
Interest at 6 per cent. for 60 days.
Interest at 7 per cent. for 52 days.
Interest at 8 per cent. for 45 days.
Interest at 9 per cent. for 40 days.
Interest at 10 per cent. for 36 days.
Interest at 12 per cent. for 30 days.
Interest at 7-30 per cent. for 50 days.
Interest at 5-20 per cent. for 70 days.
Interest at 10-40 per cent. for 35 days.
Interest at $7\frac{1}{2}$ per cent. for 48 days.
Interest at $4\frac{1}{2}$ per cent. for 80 days.

NOTE.—The figures on the left of the perpendicular line are dollars, and on the right decimals of dollars. If the dollars are less than 10 prefix a cipher.

EXAMPLE 1.—What is the interest of $120 for 15 days at 4 per cent.?

		Days.	Days.
$120=principal.		15=$\frac{1}{6}$ of	90
6)1	20 cts.=interest for 90 days.		
	20 cts.=interest for 15 days.		

EXAMPLE 2.—What is the interest of $132 for 13 days, at 7 per cent.?

		Days.	Days.
$132=principal.		13=$\frac{1}{4}$ of	52.
4)1	32 cts.=interest for 52 days.		
	33 cts.=interest for 13 days.		

EXAMPLE 3.—What is the interest of $520 for 9 days, at 8 per cent.?

		Days. Days.
$520=principal.		9=⅕ of 45.
5)5	20 cts.=interest for 45 days.	
$1	04 cts.=interest for 9 days.	

EXAMPLE 4.—What is the interest of $462 for 64 days, at 7½ per cent.?

		Days. Days. Days.
$462=principal.		64=48+16.
3)4	62 cts.=interest for 48 days.	
$1	54 cts.=interest for 16 days.	
$6	16 cts.=interest for 64 days.	

REMARK.—We have now illustrated several examples by the different per cents., and if the student will study carefully the solution to the above examples, he will in a short time be very rapid in this mode of computing interest.

NOTE.—The preceding mode of computing interest is derived and deduced from the cancelling system, as the ingenious student will readily see. It is a short and easy way of finding interest for days when the days are even or aliquot parts; but when they are not multiples, and three or four divisions are necessary, the cancelling system is much more simple and easy. We will here illustrate an example to show the difference.

Required, the interest of $420 for 49 days, at 6 per cent.:

BANKERS' METHOD.

2)4	20 cts.=int. for 60 days.
2)2	10 cts.=int. for 30 days.
5)1	05 cts.=int. for 15 days.
3)	21 cts.=int. for 3 days.
	7 cts.=int. for 1 day.
$3	43 cts.=int for 49 days.

CANCELLING METHOD.

	~~42~~~~0~~—70
~~6~~—~~3~~~~6~~	~~6~~
	49
	70

$3.430 Ans.

The cancelling method is much more brief—we simply cancel 6 in 36, and the quotient 6 into 420; there is no divisor left; hence, 70×49 gives the interest at *once.*

If the time had been 15 or 20 days, the Bankers' method would have been equally as short, because 15 and 20 are aliquot parts of 60. The superiority of the cancelling system above all others is this, it takes advantage of the *principal* as well as the *time.*

For the benefit of the student, and for the convenience of business men, we will investigate this system to its full extent, and explain how to take advantage of the *principal* when no advantage can be taken of the *days.* This is one of the most important characteristics of interest, and very often saves much labor. *It should be used when the days are not even or aliquot parts.*

The following table shows the different sums of money (at the different per cents.) that bear one cent interest a day; hence, the time in days is always the interest in cents; therefore, to find the interest on any of the following notes, at the per cent. attached to it in the table, we have the following

RULE.—*Draw a perpendicular line, cutting off the two right hand figures of the days for cents, and you have the interest for the given time.*

Interest of $90 at 4 per cent. for 1 day is 1 cent.
Interest of $72 at 5 per cent. for 1 day is 1 cent
Interest of $60 at 6 per cent. for 1 day is 1 cent
Interest of $52 at 7 per cent. for 1 day is 1 cent
Interest of $45 at 8 per cent. for 1 day is 1 cent.
Interest of $40 at 9 per cent. for 1 day is 1 cent.
Interest of $36 at 10 per cent. for 1 day is 1 cent.
Interest of $30 at 12 per cent. for 1 day is 1 cent.
Interest of $50 at 7-30 per cent. for 1 day is 1 cent.

Interest of \$70 at 5.20 per cent. for 1 day is 1 cent.
Interest of \$35 at 10.40 per cent. for 1 day is 1 cent.
Interest of \$48 at $7\frac{1}{2}$ per cent. for 1 day is 1 cent.
Interest of \$80 at $4\frac{1}{2}$ per cent. for 1 day is 1 cent.
Interest of \$24 at 15 per cent. for 1 day is 1 cent.

NOTE.—The 7-30 Government Bonds are calculated on the base of 365 days to the year, and the 5-20s and 10-40s on the base of 364 days to the year.

Problems Solved by both Methods.

We will now solve some examples by both methods to further illustrate this system, and for the purpose of teaching the pupil how to use his judgment. He will then have learned a rule *more valuable than all others.*

EXAMPLE 5.—What is the interest on \$180 for 75 days, at 6 per cent.?

Operation by taking advantage of the dollar.

75=the days. \$60×3=\$180.
\$0 | 75 cts =the interest of \$60 for 75 days.
| 3 Multiply by 3.

Ans. \$2 | 25 cts.=the interest of \$180 for 75 days.

Operation by the Bankers' method.

\$180=the principal. 60da.+15da.=75da.
4)\$1 | 80 cts.=the interest for 60 days.
| 45 cts.=the interest for 15 days.

Ans. \$2 | 25 cts.=the interest for 75 days.

By the first method we multiplied by 3, because 3×\$60=\$180. By the second method we added on $\frac{1}{4}$, because 60da.+$\frac{60}{4}$da.=75da.

N. B.—When advantage can be taken of both time and principal, if the student wishes to prove his work he can first work it by the Bankers' method, and then by taking advantage of the principal, or *vice versa.* And as the two operations are entirely different, if the same result is obtained by each, he may fairly conclude that the work is correct.

LIGHTNING METHOD OF COMPUTING INTEREST

ON ALL NOTES THAT BEAR $12 PER ANNUM, OR ANY ALIQUOT PART OR MULTIPLE OF $12.

If a note bears $12 per annum it will certainly bear $1 per month ; hence, the time in months would be the interest in dollars and the decimal parts of a dollar ; therefore, when the note bears $12 per annum we have the following

RULE.—*Reduce the years to months, add in the given months, and place one third of the days to the right of this number, and you have the interest in dimes.*

EXAMPLE 1.—Required, the interest of $200 for 3 years 7 months and 12 days, at 6 per cent.

200	$\frac{1}{3}$ of 12 days=4.
6	
———	Yr. Mo. Da.
$12.00=int. for 1 yr.	3 7 12=43.4mo.

Hence, 43.4 dimes, or $43.40 cts. Ans.

We see by inspection that this note bears $12 interest a year ; hence, the time reduced to months, with one third of the days to the right, is the interest in dimes. If this note bore $6 a year instead of $12 we would take one half of the above interest; if it bore $18 instead of $12, we would add one half; if it bore $24 instead of 12 we would multiply by 2, etc.

EXAMPLE 2.—Required, the interest of $150 for 2 years 5 months and 13 days, at 8 per cent.

150	$\frac{1}{3}$ of 13 days=4$\frac{1}{3}$.
8	
——	Yr. Da. Mo.
$12.00=int. for 1 yr.	2 5 13=29.4$\frac{1}{3}$ mos.

Hence 29.4\frac{1}{3}$ dimes, or 29.43\frac{1}{3}$ cts. Ans.

We see by inspection that this note bears $12 interest a year; hence, the time reduced to months, with one third of the days placed to the right, gives the interest at once.

EXAMPLE 3.—Required, the interest of $160 for 11 years 11 months and 11 days, at 7$\frac{1}{2}$ per cent.

160	$\frac{1}{3}$ of 11 days=3$\frac{2}{3}$.
7$\frac{1}{2}$	
——	Yr. Mo. Da.
$12.00=int. for 1 yr.	11 11 11=143.3$\frac{2}{3}$ mos.

Hence, 143.3\frac{2}{3}$ dimes, or 140.36\frac{2}{3}$ cts. Ans.

WHEN THE INTEREST IS MORE OR LESS THAN $12 A YEAR.

RULE.—*First find the interest for the given time on the base of* $12 *interest a year; then, if the interest on the note is only* $6 *a year, divide by* 2; *if* $24 *a year, multiply by* 2; *if* $18 *a year, add on one half, etc.*

EXAMPLE 1.—What is the interest of $300 for 4 years 7 months and 18 days, at 6 per cent.?

	$\frac{1}{3}$ of 18 days=6.
300	4yr. 7mo. 18da.=55.6mo.
6	
$18.00=int. for 1 year.	2)55.6, int. at $12 a year.
$18=1½ times $12.	278
	$83.4 Ans.

If the interest was $12 a year $55.60 would be the answer, because 55.6 is the time reduced to months; but it bears $18 a year, or 1½ times 12; hence, 1½ times 55.6 gives the interest at once.

Example 2.—required, the interest of $150 for 3 years 9 months and 27 days, at 4 per cent.

	$\frac{1}{3}$ of 27 days=9.
150	3yr. 9mo. 27 da.=45.9mo.
4	2)45.9, int. at $12 a year.
$6.00=int. for 1 year.	$22.95 Ans.
$6=½ times $12.	

If the interest was $12 a year $45.90 would be the answer—because 245.9 is the time reduced to months; but it bears $6 a year, or ½ times 12: hence, ½ times 45.9 gives the interest at once.

Remark.—We have now fully explained our method of computing interest at the three different bases. Any and every problem in interest can be solved by one of these three bases. Some problems can be solved easier by one base than another. Where the days are divisible by 3, and their number, annexed to the months, divisible by 12, it is the shortest and best method to use the base at one per cent. By using one or the other of these three bases the student can avoid the use of vulgar fractions. The student must study these three principles carefully, and learn to adopt readily the base best suited to the problem to be solved.

PARTIAL PAYMENTS ON NOTES, BONDS AND MORTGAGES.

To compute interest on notes, bonds and mortgages, on which partial payments have been made, two or three rules are given. The following is called the common rule, and applies to cases where the time is short, and payments made within a year of each other. This rule is sanctioned by custom and *common law;* it is true to the principles of simple interest, and requires no special enactment. The other rules are rules of *law*, made to suit such cases as require (either expressed or implied) annual interest to be paid, and, of course, apply to no business transactions closed within a year.

RULE.—*Compute the interest of the principal sum for the whole time to the day of settlement, and find the amount. Compute the interest on the several payments from the time each was paid to the day of settlement; add the several payments and the interest on each together and call the sum the amount of the payments; subtracting the amount of the payments from the amount of the principal will leave the sum due.*

EXAMPLE.—A gave his note to B for $10,000 ; at the end of 4 months A paid $6,000, and at the expiration of another 4 months he paid an additional sum of $3,000 ; how much did he owe B at the close of the year?

BY THE COMMON RULE.

Principal		$10,000
Interest for the whole time		600
Amount		$10,600
1st payment	$6,000	
Interest, 8 months	240	
2d payment	3,000	
Interest, 4 months	60	
Amount	$9,300	9,300
Due		$1,300

PROBLEMS IN INTEREST.

There are *four* parts or quantities connected with each operation in interest; these are the *Principal*, *Rate per cent.*, *Time*, *Interest* or *Amount*.

If any *three* of them are given the *other* may be found.

Principal, interest and time given to find the rate per cent.

EXAMPLE 1.—At what rate per cent. must $500 be put on interest to gain $120 in 4 years?

OPERATION.

```
$500
  .01
-----
 5.00
    4
-----
20.00)120.00(6 per cent. Ans.
      120.00
      ------
```

BY ANALYSIS.

The interest of $1 for the given time at 1 per cent. is 4 cts. $500 will be 500 times as much=500 ×.04=$20. Then, if $20 give 1 per ct., $120 will give $\frac{120}{20}$ =6 per cent.

Rule.—*Divide the given interest by the interest of the given sum at one per cent. for the given time, and the quotient will be the rate per cent. required.*

Principal, interest and rate per cent. given to find the time.

Example 2.—How long must \$500 be on interest at 6 per cent. to gain \$120 ?

OPERATION.

```
$500
  .06
------
30.00)120.00(4 years.  Ans.
      120.00
      ------
```

ANALYSIS.

We find the interest of \$1 at the given rate for one year is six cents. \$500 will be, therefore, 500 times as much $=500\times.06=\$30.00$. Now, if it take 1 year to gain \$30, it will require $\frac{120}{30}$ to gain $\$120=4$ years. Ans.

EQUATION OF PAYMENTS.

Equation of Payments is the process of finding the equalized or average time for the payment of several sums due at different times, without loss to either party.

To find the average or mean time of payment when the several sums have the same date.

Rule.—*Multiply each payment by the time that must elapse before it becomes due; then divide the sum of these products by the sum of the payments, and the quotient will be the average time required.*

Note.—When a payment is to be made down it has no product, but it must be added with the other payments in finding the average time.

Example.—I purchased goods to the amount of

\$1,200 ; \$300 of which I am to pay in 4 months, \$400 in 5 months, and 500 in 8 months. How long a credit ought I to receive if I pay the whole sum at once? Ans. 6 months.

Mo. Mo.	
$4 \times 300 = 1200$	A credit on \$300 for 4 months is the same as the credit on \$1 for 1200 months.
$5 \times 400 = 2000$	A credit on \$400 for 5 months is the same as the credit on \$1 for 2000 months.
$8 \times 500 = 4000$	A credit on \$500 for 6 months is the same as the credit on \$1 for 4000 months.
1200)7200(6 mo. 7200	Therefore, I should have the same credit as a credit on \$1 for 7200 mos.; and on \$1200, the whole sum, one twelve hundredth part of 7200 months, which is 6 months.

This rule is the one usually adopted by merchants, although not strictly correct; still, it is sufficiently accurate for all practical purposes

To find the average or mean time of payment when the several sums have different dates.

EXAMPLE.—Purchased of James Brown, at sundry times and on various terms of credit, as by the statement annexed. When is the *medium* time of payment?

Jan. 1,	a bill am'ting to	\$360,	on 3 months' credit.
Jan. 15,	do.	do. 186,	on 4 months' credit.
March 1,	do.	do. 450,	on 4 months' credit.
May 15,	do.	do. 300,	on 3 months' credit.
June 20,	do.	do. 500,	on 5 months' credit.

Ans. July 24, or in 115 days.

Due,	April 1,	\$360.	
"	May 15,	$186 \times 44 =$	8184
"	July 1,	$450 \times 91 =$	40950
"	Aug. 15,	$300 \times 136 =$	40800
"	Nov. 20,	$500 \times 233 =$	116500

1796 ÷ into)206434(114$\frac{845}{898}$ days.

We first find the time when each of the bills will become due. Then, since it will shorten the operation and bring the same result, *we take the time when the first bill becomes due*, instead of its *date*, for the *period* from which to compute the average time. Now, since April 1 is the period from which the average time is computed, no time will be reckoned on the first bill, but the time for the payment of the second bill extends 44 days beyond April 1, and we multiply it by 44.

Proceeding in the same manner with the remaining bills, we find the average time of payment to be 114 days and a fraction from April 1, or on the 24th of July.

RULE.—*Find the time when each of the sums becomes due, and multiply each sum by the number of days from the time of the earliest payment to the payment of each sum respectively; then proceed as in the last rule, and the quotient will be the average time required, in days, from the earliest payment.*

NOTE.—Nearly the same result may be obtained by reckoning the time in months.

In mercantile transactions it is customary to give a credit of from 3 to 9 months on bills of sale. Merchants, in settling such accounts as consist of various items of debit and credit for different times, generally employ the following

RULE.—*Place on the debtor or credit side such a sum (which may be called* MERCHANDISE BALANCE*) as will balance the account. Multiply the number of dollars in each entry by the number of days from the time the entry was made to the time of settlement, and the merchandise balance by the number of days for which credit* *was given. Then multiply the difference between the sum of the debit and the sum of the credit products by the interest of* $1 *for one day; this product will be the* INTEREST BALANCE.

When the sum of the debit products exceeds the sum of the credit products the interest balance is in favor of the debit side; but when the sum of the credit products exceeds the sum of the debit products it is in favor of the credit side. Now, to the merchandise balance add the interest balance, or subtract it, as the case may require, and you obtain the CASH BALANCE.

A has with B the following account:

1849.		*Dr.*	1849.		*Cr.*
Jan. 2.	To merchandise,	$200	Feb. 29.	By merchandise,	$100
April 20.	" "	400	May 10.	" "	300

If interest is estimated at 7 per cent., and a credit of 60 days is allowed on the different sums, what is the cash balance August 20, 1849? Ans. $206.54.

EXPLANATION.—Without interest the cash balance would be $200.

The object of these changes is to give the learner an accurate and complete knowledge of numbers and of division, and the result is not the only object sought for, as many young learners suppose.

How many times is 75 contained in 575? or divide 575 by 75. Ans. $7\frac{2}{3}$.

Divide 800 by $12\frac{1}{2}$. Quotient, 64.
Divide 27 by $16\frac{2}{3}$. Quotient, $1\frac{62}{100}$, or $1\frac{31}{50}$.

A person spent $6 for oranges, at $6\frac{1}{4}$ cents apiece, how many did he purchase? Ans. 96.

When two or more numbers are to be multiplied together, and one or more of them have a cipher on the right, as 24 by 20, we may take the cipher from one number and annex it to the other without affecting the product: thus 24×20 is the same as 240×2; $286 \times 1300 = 28600 \times 13$; and $350 \times 70 \times 40 = 35 \times 7 \times 4 \times 1000$, etc.

Every fact of this kind, though extremely simple, will be very useful to those who wish to be skilful in operation.

NOTE.—If there are ciphers at the right hand either of the multiplier or multiplicand, or of both, they may be neglected to the close of the operation, when they must be annexed to the product.

REMARK.—We now give a few examples for the purpose of teaching the pupil how to use his judgment; he will then have learned a rule *more valuable* than all others.

Multiplication and Division Combined.

When it becomes necessary to multiply two or more numbers together, and divide by a third, or by a product of a third and fourth, it must be *literally done if the numbers are prime.* For example:

Multiply 19 by 13 and divide that product by 7.

This must be done at full length, because the numbers are *prime,* and in all such cases there will result a *fraction.*

But in *actual business* the problems are *almost all* reducible by short operations, as the prices of articles or amount called for always corresponds with some *aliquot* part of our scale of computation; and when two or more of the numbers are *composite numbers* the work *can always* be contracted.

EXAMPLE.—Multiply 375 by 7, and divide that product by 21. To obtain the answer it is sufficient to divide 375 by 3, which gives 125. The 7 divides the 21, and the factor 3 remains for a divisor.

Here it becomes necessary to lay down a *plan of operation:* Draw a perpendicular line, and place all numbers that are to be multiplied together under each other on the right hand side, and all numbers that are divisors under each other on the left hand side.

EXAMPLES.

Multiply 140 by 36 and divide that product by 84. We place the numbers thus :

$$\begin{array}{r|l} & 140 \\ 84 & 36 \end{array}$$

We may cast out *equal factors* from each side of the line without *affecting the result.* In this case 12 will divide 84 and 36, then the numbers will stand thus :

$$\begin{array}{r|l} & 140 \\ 7 & 3 \end{array}$$

But 7 divides 140 and gives 20, which, multiplied by 3, gives 60 for the result.

Multiply 4783 by 39, and divide that product by 13.

$$\begin{array}{r|l} & 4783 \\ \cancel{13} & \cancel{39}\ 3 \end{array}$$

Three times 4783 must be the result.

Multiply 80 by 9, that product by 21, and divide the whole by the product of $60 \times 6 \times 14$.

$$\begin{array}{r|l} 3\ \cancel{60} & \cancel{80}\ 4 \\ 6 & 9 \\ 2\ \cancel{14} & \cancel{21}\ 3 \end{array}$$

In the foregoing divide 60 and 80 by 20, and 14 and 21 by 7, and those numbers will stand cancelled, with 3 and 4, 2 and 3 at their sides.

Now, the product $3 \times 6 \times 2$ on the divisor side is equal to 4 times 9 on the other, and the remaining 3 is the result.

General Rules Cancellation.

1. Draw a perpendicular line; observe this line represents the sign of equality. On the right hand side of this line place dividends only, on the left hand side place divisors only. Having placed dividends on the right and divisors on the left, as above directed,

2. Notice whether there are ciphers both on the right and left of the line; if so, erase an equal number from each side.

3. Notice whether the same number stands both on the right and left of the line; if so, erase them both.

4. Notice again if any number on either side of the line will divide any number on the opposite side without a remainder; if so, divide and erase the two numbers, retaining the quotient figure on the side of the larger number.

5. See if any two numbers, one on each side, can be divided by any assumed number without a remainder; if so, divide them by that number and retain only their quotients. Proceed in the same manner as far as practicable, then

6. Multiply all the numbers remaining on the right hand side of the line for a dividend, and those remaining on the left for a divisor.

7. Divide, and the quotient is the answer.

THE MILLER'S RULE FOR WEIGHING WHEAT.

Wheat weighing 58 pounds and upwards per bushel is considered merchantable wheat, and 60 pounds of merchantable wheat make a standard bushel. Hence, wheat weighing less than 60 pounds per bushel will lose in making up ; but, weighing more, it will gain.

When wheat weighs less than 58 pounds per bushel it is customary, on account of the inferior yield of light wheat, to take two pounds for one in making up the weight; hence, it will take 63 pounds to make up a bushel, provided the wheat weighs but 57, and 64 if the wheat weighs but 56 pounds per bushel.

Case I.—To change merchantable wheat to standard weight.

Rule.—Bring the whole quantity of wheat to pounds and divide by 60.

Example 1.—How many standard bushels of wheat are in 150 bushels, each weighing 58 pounds?

150	Or, each bushel lacks 2 lbs.; 150
58	2
1200	6,0)30,0
750	
	From 150 bush. Deficiency, 5
6,0)870,0	Take 5
Ans. 145 bush.	Leaves 145, the answer.

2. How many standard bushels of wheat are in 80 bushels 45 pounds, weighing 63 ?

Bush.	lbs.		
80	45	Or, 80 bus.	
63		3	
285		6,0)24,0	excess of w'gt.
480		4 bus.	
6,0)508,5		80	45 lbs.

Ans. 84 bu. 45 lbs.$=84\frac{3}{4}$ bu. Ans. 84 bu. 45 lbs. or 3 p.

3. How many standard bushels of wheat are in 175 bushels 37 pounds, weighing 59 ? Ans. 172 bus. 42 lbs.

4. How many standard bushels are in 100 bushels 15 pounds, weighing 62 pounds per bushel? Ans. 103 bus. 35 lbs.

Case II.—When wheat weighs less than 58.

Rule.—*Bring the whole quantity to pounds, and divide by as many pounds as make a standard bushel of such wheat.*

Example 1.—How many bushels of good wheat are equal to 100 bushels weighing 57 ?

```
   100                     Or, 6 lbs. per bus.=600 lbs.
    57                       63)600(9 bus. 33 lbs. defect.
   ----                         567
63)5700(90 bus. 30 lbs.        ----
   567         From 100 bus.    33
   ----        Take   9 33
    30 lbs.         -------
                Ans. 90 30
```

NOTE.—The odd pounds in the above and following results are also subject to a small drawback, viz., 1 lb. in every 21 when the wheat weighs 57; 1 in 16 when it weighs 56, and so on; consequently the above ought, in strictness, to be 90 bushels, and rather more than $28\frac{1}{2}$ pounds, but millers seldom make this deduction.

2. How many standard bushels of merchantable wheat will be equal to 250 bushels 18 lbs. weighing 56 lbs. per bushel? Ans. 219 bus. 2 lbs.

3. How much good wheat is equal to 1000 bushels weighing 55? Ans. 846 bus. 10 lbs.

NOTE.—Before dismissing this rule it appears proper that a few remarks should be made, in order to show the young farmer the importance of understanding it properly. There are different methods of "making up wheat" (*i. e.*, finding its merchantable value), and these methods give different results; hence the necessity of the subject being understood by all concerned. I shall not undertake to determine between the farmer and the miller which is or which is not the fair way; but, after explaining the principle, leave them to make their bargains as they may choose.

If I have a bushel of wheat that weighs but 57 pounds, then six pounds of the *same kind of wheat* will be necessary to make this a merchantable bushel, so that 63 pounds of this quality of wheat will make a standard bushel; and it is upon this supposition that the preceding calculations are founded. But a number of millers use a method of calculation by which they take *good* wheat for the odd pounds—*i. e.*, they take a bushel full, say 57 pounds, of the wheat they are measuring, and instead of taking six pounds more of *the same kind* to make it up, they take six pounds of good or *merchantable wheat.* Their method of calculation is as follows:

Required, the good wheat in 1000 bushels weighing 55? Defect, 10 lbs. per bus.

```
From 1000 bus.                      1000
Take  166 40                          10
                                  -------
                                 6,0)1000,0
     --------                    ----------
Ans. 833 .20                     Bush. 166 40
```

We see that this gives a result nearly 13 bushels more in the miller's favor than the former method; and this I know to be practiced by many.

There is another method sometimes used by those who are not very scrupulous in their distinctions between right and wrong. They find the whole defect in pounds, and divide by the weight of a bushel of the wheat to find how many bushels of *that kind* of wheat will make up the defect, thus:

Required, the good wheat in 1000 bushels weighing 55 pounds per bushel?

Defect, 10 lbs. per bushel=10000 in all.

```
From 1000 bus.       55)10000(181 bus. 45 lbs.
Take  181 45            55
     -------            ---
Ans. 818 10             450
                        440
                        ---
                         100
                          55
                          --
                          45 lbs.
```

We see that this method gives 15 bushels more to the miller than the last and 28 more than the first. It, however, shows how much of the same kind of wheat must be added to the 1000 to make 1000 bushels of good wheat, viz., 181 bushels 45 pounds—for 1181 bushels 45 pounds, weighing 55, will just make 1000 bushels "made up weight." This method would be as erroneous as calculating discount by the rule for interest.

Another method is to take two pounds for one up to 58, and pound for pound afterwards. To do this bring the whole quantity to pounds, and if the wheat weigh 57 divide by 61; if 56, by 62, and so on. This appears more reasonable than the others, as it makes less difference between wheat barely merchantable and that which is not quite so. By the former rule, if the wheat weigh 58, two pounds per bushel will make it up, but if 57, six pounds are necessary; by this rule only one extra pound would be taken. In subtracting the odd pounds, the lower number being greatest (suppose the wheat to weigh, say 57), the calculator may be at a loss whether to take from 57, 60 or 63. In this case, let the running weight be what it may, we should take from the weight made up, as in example 1, case 2, we took from 63.

This subject is of importance to both farmers and millers, and if they do not attend to it they deserve to be cheated.

SQUARE AND CUBE ROOTS.

A number multiplied by itself once will produce the square of that number.

A number multiplied by itself twice will produce the cube of that number.

Extracting the root of a number is finding what number multiplied into itself, once for square and twice for cube, will produce the given number.

Numbers possess the following properties:

1. A square multiplied by a square will produce a square.

2. A square divided by a square the quotient will be a square.

3. A cube multiplied by a cube will produce a cube.

4. A cube divided by a cube the quotient will be a cube.

5. If the unit figure of a square number is 5 we may multiply by the square number 4, and we shall produce a square whose unit period will be ciphers.

6. If the unit figure of a cube is 5 we may multiply by the cube number 8, and produce a cube whose unit period will be ciphers.

The following table should be committed to memory, in order to be able to *extract roots immediately by inspection:*

Numbers.	1	2	3	4	5	6	7	8	9	10
Squares.	1	4	9	16	25	36	49	64	81	100
Cubes.	1	8	27	64	125	216	343	512	729	1000

To Extract Square Root by Inspection.

Commencing at the right hand, point the number off into periods of two figures each, and the root will contain as many figures as there are periods. From the table find the greatest square of each period, and these numbers, set down in their order, will be the answer required.

EXAMPLE 1.—What is the square root of 225? Ans. 15.

Pointing it off (2.25) we find it consists of two periods, 2 and 25. By referring to the table we find that the greatest square in 2 is 1, and the square in 25 is 5=15.

2. What is the square root of 2025? Ans. 45.

The greatest square in 20 is 4, and 5 is the only number whose square is 5 in the unit's place.

3. Extract the square root of 6561. Ans. 81.

As the unit figure is 1, and in the table we find 1 only at 1 and 81, we will divide 6561 by 81, and we find the quotient 81. 81 is, therefore, the square root.

4. Extract the square root of 106729. Ans. 327.

As the unit figure is 9, if the number is a square it must either divide by 9 or 49. Dividing by 9 we obtain 11881, which is a prime number=109. This number multiplied by 3, the root of 9, gives 327.

5. Extract the square root of 451584. Ans. 672.

As the unit figure is 4, and in the table of squares we find 4 only at 4 and 64, the number, if a square, must divide by 4 or 64, or both. Dividing by 4, we have the factors 4 and 112896. This last factor ends in 6; therefore, by the table, we see it must divide by 16 or 36. Dividing by 36 we have the factors 36 and 3136. This last factor, ending in 6, we divide by 16, and have the factors 16 and 196. Dividing this last factor by 4 we have 4 and 49. Take our divisors and last factor, 49, and we have for the original number, $4 \times 36 \times 16 \times 4 \times 49$, the roots of which are $2 \times 6 \times 4 \times 2 \times 7 = 672$.

6 Extract the square root of 1225. Ans. 35.

Divide by the square number 25 and we have the two factors 25 and 49 as equivalent to the given number. Roots of these factors, $5\times7=35$. Or, as the number ends in 25, we may multiply by 4 and we have 4900, the root of which is 70—which, divided by 2, the square root of 4, gives 35.

PROBLEM.—The top of a castle is 45 yards high and the castle is surrounded by a ditch 60 yards wide. Required, the length of a rope that will reach from the outside of the ditch to the top of the castle. Ans. 75 yards.

The usual rule for this is to add the square of the two sides and extract the square root of the sum; but so much labor is never necessary when the two numbers have a common divisor, or when the answer is a composite number.

Take 45 and 60 and divide them both by 15 and they will be reduced to 3 and 4; square these, and their sum $9+16=25$; extract the square root of 25, which is 5, multiplied by 15 gives 75.

When it is requisite to multiply several numbers together and extract the root of their product, try to change them into *factors* and extract the root *before multiplication.*

To Extract Cube Root by Inspection.

We can extract the root of cube numbers by inspection when they do not contain more than two periods of three figures each.

By examining the table it will appear evident that if the unit figure of the power be 1 the unit figure in the root will be 1; if it be 8 the root will be 2; if 7 it will be 3; if 4 the root will be 4; if 5 it will be

5; if 6 it will be 6; if 3 it will be 7; if 2 it will be 8; and if it be 9 the root will be 9.

EXAMPLE 1.—Find the cube root of 117649.

This number consists of two periods. Compare the *ten's* period with the cubes in the table and we find that 117 lies between 64 and 125. The cube root of the tens then must be 4. The unit figure of the unit period being 9 the root must be 9; therefore 49 is the root required.

2. What is the cube root of 389017? Ans. 73.

By looking in the table we find that the ten's period, 389, lies between 343 and 512; the root must, therefore, be 7. The unit figure of the unit period being 7 the root must be 3, therefore 73 is the required root.

When a cube has more than two periods it can generally be reduced to two by dividing by some cube number, unless the root is a prime number.

3. What is the cube root of 3241792. Ans. 148.

As the unit figure of the unit period is 2 the root of that period must be 8. Now, dividing 3241792 by 8, we have 405224, a number consisting of but two periods, the root of which we find by inspection to be 74—which, multiplied by 2, the root of 8, gives 148, the root required.

4. What is the cube root of 12977875? Ans. 235.

As this number ends in 5 we will multiply it by 8, and if it is a cube the unit period will be ciphers: $12977875 \times 8 = 103623000$. This number may be regarded as two periods—the unit period being 0. By inspection the root of 103623 is 47; annexing the cipher equals 470—which, divided by 2, the root of 8, gives 235, the answer.

To find the cube root of surds, very nearly:

RULE.—*Take the nearest cube to the given number and call it the assumed cube. Double the assumed cube and add the number to it; also, double the number and add the assumed cube to it. Take the difference of these sums, then say, as double the assumed cube added to the number is to this difference, so is the assumed root to the correction. This correction, added to or subtracted from the assumed root, as the case may require, will give the cube root, very nearly.*

5. What is the cube root of 214?

The nearest cube to 214 is 216, the root of which is 6, and it is evident the root of 214 will be less, and, to state the proportion according to the rule, we have

$$\begin{array}{rr} 216\times2=432 & 214\times2=428 \\ 214 & 216 \\ \hline 646 & 644=2 \text{ difference.} \end{array}$$

As 646 : 2 :: 6 : to correction—

$$2\times6=12. \quad \begin{array}{r} 646)12.00000(0.01857+\text{correction.} \\ 646 \\ \hline 5540 \\ 5168 \\ \hline 3720 \\ 3230 \\ \hline 4900 \\ 4522 \end{array}$$

As the assumed root is more, we subtract the correction:

$$\begin{array}{l} 6.00000 \\ 0.01857+ \\ \hline 5.98143+\text{the required root, very nearly.} \end{array}$$

A TABLE FOR MEASURING TIMBER.

Quarter Girt.	Area.	Quarter Girt.	Area.	Quarter Girt.	Area.
Inches.	Feet.	Inches.	Feet.	Inches.	Feet.
6	.250	12	1.000	18	2.250
6¼	.272	12¼	1.042	18½	2.376
6½	.294	12½	1.085	19	2.506
6¾	.317	12¾	1.129	19½	2.640
7	.340	13	1.174	20	2.777
7¼	.364	13¼	1.219	20½	2.917
7½	.390	13½	1.265	21	3.062
7¾	.417	13¾	1.313	21½	3.209
8	.444	14	1.361	22	3.362
8¼	.472	14¼	1.410	22½	3.516
8½	.501	14½	1.460	23	3:673
8¾	.531	14¾	1.511	23½	3.835
9	.562	15	1.562	24	4.000
9¼	.594	15¼	1.615	24½	4.168
9½	.626	15½	1.668	25	4.340
9¾	.659	15¾	1.722	25½	4.516
10	.694	16	1.777	26	4.694
10¼	.730	16¼	1.833	26½	4.876
10½	.766	16½	1.890	27	5.062
10¾	.803	16¾	1.948	27½	5.252
11	.840	17	2.006	28	5.444
11¼	.878	17¼	2.066	28½	5.640
11½	.918	17½	2.126	29	5.840
11¾	.959	17¾	2.1$6	29½	6.044
				30	6.250

Rule.—(By the Carpenters' Rule.)—*Measure the circumference of the piece of timber in the middle and take a quarter of it in inches; call this the girt. Then set* 12 *on* D *to the length in feet on* C, *and against the girt in inches on* D *you will find the content in feet on* C.

Example.—If a piece of round timber be 18 feet long, and the quarter girt 24 inches, how many feet of timber are contained therein?

```
      24 quarter girt.
      24
      ——
      96
     48
     ———
     596 square.
      18
    ————
    4608
    576
   —————
144)10368(72 feet.
    1008
    ————
     288
     288
```

BY THE TABLE.

Against 24 stands	4.00
Length,	18
Product,	72.00

Ans. 72 feet.

By the Carpenters' Rule.—12 on D : 18 on C : 24 on D : 72 on C.

Problem I.—To find the solid contents of squared or four-sided timber by the Carpenters' Rule—as 12 on D : length on C : quarter girt on D : solidity on C.

Rule I.—*Multiply the breadth in the middle by the depth in the middle, and that product by the length for the solidity.*

Note.—If the tree taper regularly from one end to the other half the sum of the breadths of the two ends will be the breadth in the middle, and half the sum of the depths of the two ends will be the depth in the middle.

Rule II.—*Multiply the sum of the breadths of the two ends by the sum of the depths, to which add the product of the breadth and depth of each end; one sixth of this sum multiplied by the length will give the correct solidity of any piece of squared timber tapering regularly.*

Problem II.—To find how much in length will make a solid foot, or any other assigned quantity of squared timber, of equal dimensions from end to end.

Rule.—*Divide* 1728, *the solid inches in a foot, or the solidity to be cut off, by the area of the end in inches, and the quotient will be the length in inches.*

Note.—To answer the purpose of the above rule some carpenters' rules have a little table upon them in the following form, called a *table of timber measure:*

0	0	0	0	9	0	11	3	9	inches.
144	36	16	9	5	4	2	2	1	feet.
1	2	3	4	5	6	7	8	9	side of the square.

This table shows that if the side of the square be one inch the length must be 144 feet; if two inches be the side of the square the length must be 36 feet to make a solid foot.

Problem III.—To find the solidity of round or unsquared timber.

Rule I.—*Gird the timber round the middle with a* *string; one fourth part of this girt squared and multiplied by the length will give the solidity.*

NOTE.—If the circumference be taken in inches and the length in feet, divide the last product by 144.

RULE II.—(BY THE TABLE).—*Multiply the area corresponding to the quarier girt in inches by the length of the piece of timber in feet, and the product will be the solidity.*

NOTE.—If the quarter girt exceed the table take half of it, and four times the content thus formed will be the answer.

How do you do when the timber tapers?

Gird the timber at as many points as may be necessary, and divide the sum of the girts by their number for the mean girt, of which take one fifth and proceed as before.

If a tree, girting 14 feet at the thicker end and 2 feet at the smaller end, be 24 feet in length, how many solid feet will it contain? Ans. 122.88.

A tree girts at five different places as follows: in the first 9.43 feet, in the second 7.92 feet, in the third 6.15 feet, in the 4th 4.74 feet, and in the fifth 3.16 feet; now, if the length of the tree be 17.25 feet, what is its solidity? Ans. 54.42499 cubic feet.

OF LOGS FOR SAWING.

What is often necessary for lumber merchants?

It is often necessary for lumber merchants to ascertain the number of feet of boards which can be cut from a given log; or, in other words, to find how many logs will be necessary to make a given amount of boards.

What is a standard board?

A standard board is one which is 12 inches wide, one inch thick and 12 feet long; hence, a standard board is one inch thick and contains 12 square feet.

What is a standard saw log?

A standard log is 12 feet long and 24 inches in diameter.

How will you find the number of feet of boards which can be sawed from a standard log?

If we saw off, say two inches from each side, the log will be reduced to a square 20 inches on a side. Now, since a standard board is one inch in thickness, and since the saw cuts about one quarter of an inch each time it goes through, it follows that one fourth of the log will be consumed by the saw. Hence we shall have $20 \times \frac{3}{4}$=the number of boards cut from the log. Now, if the width of a board in inches be divided by 12, and the quotient be multiplied by the length in feet, the product will be the number of square feet in the board. Hence, $\frac{20}{12} \times$ length of log in feet=the square feet in each board. Therefore, $20 \times \frac{3}{4} \times \frac{20}{12} \times$ length of log=the square feet in all the boards$=20 \times 10 \times \frac{3}{4} \times \frac{2}{12} \times$ length of log $= 20 \times 10 \times \frac{1}{8} \times$ length. And the same may be shown for a log of any length.

What, then, is the rule for finding the number of feet of boards which can be cut from any log whatever?

From the diameter of the log in inches subtract four for the slabs; then multiply the remainder by half itself and the product by the length of the log in feet and divide the result by eight; the quotient will the number of square feet.

EXAMPLE 1.—What is the number of feet of boards which can be cut from a standard log ?

Diameter,	24 inches,
For slabs,	4
Remainder,	20
Half remainder,	10
	200
Length of log,	12
	8)2400
	300=the number of feet.

2. How many feet can be cut from a log 12 inches in diameter and 12 feet long ? Ans. 48.

3. How many feet can be cut from a log 20 inches in diameter and 16 feet long ? Ans. 256.

4. How many feet can be cut from a log 24 inches in diameter and 16 feet long ? Ans. 400.

5. How many feet can be cut from a log 28 inches in diameter and 14 feet long ? Ans. 504.

CARPENTERS AND JOINERS' WORK.

In what does carpenters and joiners' work consist?

Carpenters and joiners' work is that of flooring, roofing, etc., and is generally measured by the square of 100 square feet.

When is a roof said to have a true pitch?

In carpentry a roof is said to have a *true pitch* when the length of the rafters is three fourths the breadth of the building. The rafters then are nearly at right angles. It is, therefore, customary to take once and a half times the area of the flat of the building for the area of the roof.

EXAMPLE 1.—How many squares, of 100 square feet each, in a floor 48 feet 6 inches long and 24 feet 3 inches broad? Ans. 11 and $76\frac{1}{8}$ sq. ft.

2. A floor is 36 feet 6 inches long and 16 feet 6 inches broad, how many squares does it contain? Ans. 5 and $98\frac{1}{8}$ sq. ft.

3. How many squares are there in a partition 91 feet 9 inches long and 11 feet 3 inches high? Ans. 10 and 32 sq. ft.

4. If a house measure within the walls 52 feet 8 inches in length and 30 feet 6 inches in breadth, and the roof be of the true pitch, what will the roofing cost at $1.40 per square? Ans. $33.733.

Of Bins for Grain.

What is a bin?

It is a wooden box used by farmers for the storage of their grain.

Of what are bins generally made?

Their bottoms or bases are generally rectangles and horizontal and their sides vertical.

How many cubic feet are there in a bushel?

Since a bushel contains 2150.4 cubic inches, and a cubic foot 1728 inches, it follows that a bushel contains $1\frac{1}{4}$ cubic feet, nearly.

Having any number of bushels, how then will you find the corresponding number of cubic feet?

Increase the number of bushels one fourth itself, and the result will be the number of cubic feet.

EXAMPLE 1.—A bin contains 372 bushels; how many cubic feet does it contain?

$372 \div 4 = 93$; hence, $372 + 93 = 465$ cubic feet.

2. In a bin containing 400 bushels how many cubic feet? Ans. 500.

How will you find the number of bushels which a bin of a given size will hold?

Find the content of the bin in cubic feet, then diminish the content by one fifth, and the result will be the content in bushels.

3. A bin is 8 feet long, 4 feet wide and 5 feet high; how many bushels will it hold?

$8 \times 4 \times 5 = 160$

then, $160 \div 5 = 32$: $160 - 32 = 128$ bushels = capacity of bin.

4. How many bushels will a bin contain which is

7 feet long, 3 feet wide and 6 feet in height. Ans. 100.8 bush.

How will you find the dimensions of a bin which shall contain a given number of bushels?

Increase the number of bushels one fourth itself and the result will show the number of cubic feet which the bin will contain. Then, when the two dimensions of the bin are known, divide the last result by their product, and the quotient will be the other dimension.

5. What must be the height of a bin that will contain 600 bushels, its length being 8 feet and its breadth 4?

600÷4=150; hence, 600+150=750=the cubic feet, and 8×4=32, the product of the given dimensions. Then, 750÷32=23.44 feet the height of the bin.

6. What must be the width of a bin that shall contain 900 bushels, the height being 12 and the length 10 feet?

900÷4=225; hence, 900+225=1135=the cubic feet; and 12×10=120, the product of the given dimensions. Then, 1125÷120=9.375 feet, the width of the bin.

7. The length of a bin is 4 feet, its breadth 5 feet 6 inches; what must be its height that it may contain 136 bushels? Ans. 7 ft. 8 in.+

8. The depth of a bin is 6 feet 2 inches, the breadth 4 feet 8 inches; what must be the length that it may contain 200 bushels? Ans. 104 in.+

SLATERS AND TILERS' WORK.

How is the content of a roof found?

In this work the content of the roof is found by multiplying the length of the ridge by the girt from eave to eave. Allowances, however, must be made for the double rows of slate at the bottom.

Example 1.—The length of a slated roof is 45 feet 9 inches, and its girt 34 feet 3 inches; what is its content? Ans. 1566.9375 sq. ft.

2. What will the tiling of a barn cost at $3.40 per square of 100 feet, the length being 43 feet 10 inches and breadth 27 feet 5 inches on the flat, the eave board projecting 16 inches on each side and the roof being of the true pitch? Ans. $65.26.

BRICKLAYERS' WORK.

In how many ways is artificers' work computed?

Artificers' work in general is computed by three different measures, viz:

1st. The linear measure; or, as it is called by mechanics, running measure.

2d. Superficial or square measure, in which the computation is made by the square foot, square yard, or by the square containing 100 square feet or yards.

3d. By the cubic or solid measure, when it is estimated by the cubic foot or the cubic yard. The work, however, is often estimated in square measure, and the materials for construction in cubic measure.

What proportion do the dimensions of a brick bear to each other?

The dimensions of a brick generally bear the following proportions to each other, viz:

Length=twice the width, and
Width=twice the thickness;

and, hence, the length is equal to four times the thickness.

What are the common dimensions of a brick? How many cubic inches does it contain?

The common length of a brick is 8 inches, in which case the width is 4 inches and the thickness 2 inches. A brick of this size contains $8\times4\times2=64$ cubic inches; and since a cubic foot contains 1728 cubic inches, we have $1728\div64=27$ the number of bricks in a cubic foot.

If a brick is 9 inches long, what will be its width and what its content?

If the brick is 9 inches long, then the width is $4\frac{1}{2}$ inches, and the thickness $2\frac{1}{4}$; and then each brick will contain $9\times4\frac{1}{2}\times2\frac{1}{4}=61\frac{1}{8}$ cubic inches; and $1728\div91\frac{1}{8}=19$ nearly, the number of bricks in a cubic foot. In the examples which follow we shall suppose the brick to be 8 inches long.

How do you find the number of bricks required to build a wall of given dimensions?

1st. Find the content of the wall in cubic feet.

2d. Multiply the number of cubic feet by the number of bricks in a cubic foot, and the result will be the number of bricks required.

EXAMPLE 1.—How many bricks, of 8 inches in length, will be required to build a wall 30 feet long, a brick and a half thick and 15 feet in height? Ans. 12150.

2. How many bricks, of the usual size, will be required to build a wall 50 feet long, 2 bricks thick and 36 feet in height? Ans. 64800.

What allowance is made for the thickness of the mortar?

The thickness of mortar between the courses is nearly a quarter of an inch, so that four courses will give nearly one inch in height. The mortar, therefore, adds nearly one eighth to the height; but as one eighth is rather too large an allowance, we need not consider the mortar which goes to increase the length of the wall.

3. How many bricks would be required in the first and second examples, if we make the proper allowance for mortar? Ans. 1st. 10631¼. 2d. 56700.

How do bricklayers generally estimate their work?

Bricklayers generally estimate their work at so much per thousand bricks.

4. What is the cost of a wall 60 feet long, 20 feet high and 2½ bricks thick at $7.50 per thousand—which price we suppose to include the cost of the mortar?

If we suppose the mortar to occupy a space equal to one eighth the height of the wall, we must find the quantity of bricks under the supposition that the wall was 17½ feet in height. Ans. $354.37½.

In estimating the bricks for a house what allowances are made?

In estimating the bricks for a house, allowance must be made for the windows and doors.

Of Cisterns.

What are cisterns?

Ciscerns are large reservoirs constructed to hold water; and, to be permanent, should be made either of brick or masonry. It frequently occurs that they are to be so constructed as to hold given quantities of water, and then it becomes a useful and practical problem to calculate their exact dimensions.

How many cubic inches in a hogshead?

A hogshead contains 63 gallons, and a gallon contains 231 cubic inches; hence, 231×63×14553, the number of cubic inches in a hogshead.

How do you find the number of hogsheads which a cistern of given dimensions will contain?

1st. Find the solid content of the cistern in cubic inches.

2d. Divide the content so found by 14553 and the quotient will be the number of hogsheads.

Example.—The diameter of a cistern is 6 feet 6 inches, and height 10 feet; how many hogsheads does it contain?

The dimensions reduced to inches are 78 and 120; then, the content in cubic inches, which is 573404.832, gives

573404.832÷14553=39.40 hogsheads, nearly.

If the height of a cistern be given how do you find the diameter, so that the cistern shall contain a given number of hogsheads?

1st. Reduce the height of the cistern to inches, and the content to cubic inches.

2d. Multiply the height by the decimal .7854.

3d. Divide the content by the last result and extract the square root of the quotient, which will be the diameter of the cistern in inches.

EXAMPLE.—The height of a cistern is 10 feet; what must be its diameter that it may contain 40 hogsheads? Ans. 78.6 in. nearly.

If the diameter of a cistern be given how do you find the height, so that the cistern shall contain a given number of hogsheads?

1st. Reduce the content to cubic inches.

2d. Reduce the diameter to inches, and then multiply its square by the decimal .7854.

3d. Divide the content by the last result, and the quotient will be the height in inches.

EXAMPLE.—The diameter of a cistern is 8 feet; what must be its height that it may contain 150 hogsheads? Ans. 25 ft. 1 in., nearly.

MASONS' WORK.

What belongs to MASONRY, *and what measures are used?*

All sorts of stone work. The measure made use of is either superficial or solid.

Walls, columns, blocks of stone or marble are measured by the cubic foot; and pavements, slabs, chimney pieces, etc., are measured by the square or superficial foot. Cubic or solid measure is always used for the materials, and the square measure is sometimes used for the workmanship.

Example 1.—Required, the solid content of a wall 53 feet 6 inches long, 12 feet 3 inches high and 2 feet thick. Ans. $1310\frac{3}{4}$ ft.

2. What is the solid content of a wall, the length of which is 24 feet 3 inches, height 10 feet 9 inches, and thickness 2 feet? Ans. 521.375 ft.

3. In a chimney-piece we find the following dimensions:

Length of the mantel and slab,	4	feet	2	inches.
Breadth of both together,	3	"	2	"
Length of each jam,	4	"	4	"
Breadth of both,	1	"	9	"

Required, the superficial content. Ans. 31 ft. 10′.

PLASTERERS' WORK.

How many kinds of plasterers' work are there, and how are they measured?

Plasterers' work is of two kinds, viz: ceiling, which is plastering on laths; and rendering, which is plastering on walls. These are measured separately.

The contents are estimated either by the square foot, the square yard, or the square of 100 feet.

Inriched mouldings, etc., are rated by the running or lineal measure.

In estimating plastering, deductions are made for chimneys, doors, windows, etc.

Example 1.—How many square yards are contained in a ceiling 43 feet 3 inches long and 25 feet 6 inches broad? Ans. $122\frac{1}{2}$, nearly.

2. What is the cost of ceiling a room 21 feet 8 inches by 14 feet 10 inches, at 18 cents per square yard? Ans. \$6.42½.

3. The length of a room is 14 feet 5 inches, breadth 13 feet 2 inches, and height to the under side of the cornice 9 feet 3 inches. The cornice girts 8½ inches and projects 5 inches from the wall on the upper part next the ceiling, deducting only for one door 7 feet by 4; what will be the amount of the plastering?

Ans. { 53 yds. 5 ft. 3′ 6″ of rendering.
18 yds. 5 ft. 6′ 4″ of ceiling.
37 ft. 10′ 9″ of cornice.

How is the area of the cornice found in the above examples?

The mean length of the cornice both in the length and breadth of the house is found by taking the middle line of the cornice. Now, since the cornice projects 5 inches at the ceiling, it will project 2½ inches at the middle line; and, therefore, the length of the middle line along the length of the room will be 14 feet, and across the room 12 feet 9 inches. Then multiply the double of each of these numbers by the girth, which is 8½ inches, and the sum of the products will be the area of the cornice.

PAINTERS' WORK.

How is painters' work computed, and what allowances are made?

Painters' work is computed in square yards. Every part is measured where the color lies, and the measuring line is carried into all the mouldings and cornices.

Windows are generally done at so much a piece. It is usual to allow double measure for carved mouldings, etc.

Example 1.—How many yards of painting in a room which is 65 feet 6 inches in perimeter and 12 feet 4 inches in height? Ans. $89\frac{41}{54}$ sq. yds.

2. The length of a room is 20 feet, its breadth 14 feet 6 inches, and height 10 feet 4 inches; how many yards of painting are in it—deducting a fire-place of 4 feet by 4 feet 4 inches, and two windows, each 6 feet by 3 feet 2 inches. Ans $73\frac{2}{27}$ sq. yds.

PAVERS' WORK.

How is pavers' work estimated?

Pavers' work is done by the square yard, and the content is found by multiplying the length and breadth together.

Example 1.—What is the cost of paving a side-walk, the length of which is 35 feet 4 inches and breadth 8 feet 3 inches, at 54 cents per square yard? Ans. $17.48 9.

2. What will be the cost of paving a rectangular court yard, whose length is 63 feet and breadth 45 feet, at 2*s.* 6*d.* per square yard—there being, however, a walk running lengthwise 5 feet 3 inches broad which is to be flagged with stone costing 3*s.* per square yard? Ans. £40 5*s.* $10\frac{1}{2}$*d.*

PLUMBERS' WORK.

Plumbers' work is rated at so much a pound, or else by the hundred weight. Sheet lead, used for gutters, etc., weighs from 6 to 12 pounds per square foot. Leaden pipes vary in weight according to the diameter of their bore and thickness.

The following table shows the weight of a square foot of sheet lead, according to its thickness; and the common weight of a yard of leaden pipe, according to the diameter of the bore:

THICKNESS OF LEAD.	POUNDS TO A SQUARE FOOT.	BORE OF LEADEN PIPES.	POUNDS PER YARD.
INCH. $\frac{1}{10}$	5.899	INCH. $0\frac{3}{4}$	10
$\frac{1}{9}$	6.554	1	12
$\frac{1}{8}$	7.373	$1\frac{1}{4}$	16
$\frac{1}{7}$	8.427	$1\frac{1}{2}$	18
$\frac{1}{6}$	9.831	$1\frac{3}{4}$	21
$\frac{1}{5}$	11.797	2	24

EXAMPLE 1.—What weight of lead of $\frac{1}{10}$ of an inch in thickness will cover a flat 15 feet 6 inches long and 10 feet 3 inches broad, estimating the weight at 6 lbs. per square foot? Ans. 8 cwt. 2 qr. $1\frac{1}{4}$ lb.

2. What will be the cost of 130 yards of leaden pipe of an inch and a half bore, at 8 cents per pound, supposing each yard to weigh 18 lbs.? Ans. $187.20.

3. The lead used for a gutter is 12 feet 5 inches long and 1 foot 3 inches broad, what is its weight, supposing it to be $\frac{1}{8}$ of an inch in thickness? Ans. 101 lbs. 12 oz. 13.6 dr.

4. What is the weight of 96 yards of leaden pipe of an inch and a quarter bore? Ans. 13 cwt. 2 qr. 24 lbs.

5. What will be the cost of a sheet of lead 16 feet 6 inches long and 10 feet 4 inches broad at 5 cents per pound; the lead being $\frac{1}{6}$ of an inch in thickness? Ans. 83.81.

PERPETUAL CALENDAR.

To tell on what day of the week any date will transpire for the period of three thousand years from the Christian Era.

TABLE OF CENTENNIAL RATIOS.

200, 900, 1800, 2200, 2600, 3000,	ratio	is	0
300, 1000,	"	"	6
400, 1100, 1900, 2300, 2700,	"	"	5
500, 1200, 1600, 2000, 2400,	"	"	4
600, 1300,	"	"	3
700, 1400, 1700, 2100, 2500, 2900,	"	"	2
100, 800, 1500,	"	"	1

TABLE OF MONTHLY RATIOS.

Ratio of January is	3	Ratio of July is	2
" " February,	6	" " August,	5
" " March,	6	" " September,	1
" " April,	2	" " October,	3
" " May,	4	" " November,	6
" " June,	0	" " December,	1

Remark.—In Leap Year the ratio in January is 2, and of February, 5. The ratio of the other months remain the same.

Explanation.—To the given year add its fourth part, rejecting the fractions. To this sum add the day of the month, then add the ratio of the century and the ratio of the month. Divide this sum by 7; the remainder is the day of the week, counting Sunday as the first, Monday the second, etc. Of course, Saturday being the seventh day the remainder will be a cipher.

EXAMPLE 1.—Required, the day of the week for the 4th of July, 1870:

To the given year, which is	70
Add its fourth part, rejecting fractions, . .	17
Add the day of the month, which is . .	4
Add the ratio of the century, 1800, which is	0
Add the ratio of the month, July, which is .	2
Divide by	7)93
	13–2

We have 2 remainder, or the 2d day of the week, which is Monday.

2. The Declaration of Independence was signed July 4th, 1776. What was the day of the week?

To the given year, which is	76
Add its fourth part, rejecting fractions, . .	19
Add the day of the month, which is . . .	4
Add the ratio of the century, 1700, which is	2
Add the ratio of the month, July, which is .	2
Divide by . . -	7)103
	14–5

We have 5 remainder, or Thursday, the 5th day of the week.

3. Upon what day of the week will happen the 1st of January, or New Years, 2000?

To the given year, which is	00
Add its fourth part, which is	0
Add the day of the month, which is . . .	1
Add the ratio of 2000, which is	4
Add the ratio of January, being leap year, is	2
Divide by	7)7
	1–0

The cipher remaining gives Saturday as the answer.

4. On what day were you born?

From / To	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.
January..........	365	31	59	90	120	151	181	212	243	273	304	334
February........	334	365	28	59	89	120	150	181	212	242	273	303
March...........	306	337	365	31	61	92	122	153	184	214	245	275
April............	275	306	334	365	30	61	91	122	153	183	214	244
May.............	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	334	365	30	61	92	122	153	183
July.............	184	215	243	274	304	335	365	31	62	92	123	153
August..........	153	184	212	243	273	304	334	365	31	61	92	122
September.......	122	153	181	212	242	273	303	334	365	30	61	91
October..........	92	123	151	182	212	243	273	304	335	365	31	61
November.......	61	92	120	151	181	212	242	273	304	334	365	30
December.......	31	62	90	121	151	182	212	243	274	304	335	365

Table showing Difference of Time at 12 O'clock (Noon) at New York.

New York....12.00	M.		Boston......12.12	P. M.
Buffalo.......11.40	A. M.		Quebec......12.12	"
Cincinnati....11.18	"		Portland.... 12.15	"
Chicago......11.07	"		London....... 4.55	"
St. Louis.....10.55	"		Paris........ 5.05	"
San Francisco. 8.45	"		Rome........ 5.45	"
New Orleans ..10.56	"		Constantin'ple 6.41	"
Washington...11.48	"		Vienna....... 6.00	"
Charleston....11.36	"		St. Petersb'g. 6.57	"
Havana.......11.25	"		Pekin 12.40	A. M.

WEIGHTS AND MEASURES.

Troy Weight.

By this weight gold, silver, platina and precious stones (except diamonds) are estimated.

20 mites....1 grain.	20 pennyw'ts....1 ounce.
20 grains....1 pennyw't.	12 ounces.......1 pound.

Any quantity of gold is supposed to be divided into 24 parts, called *carats*. If pure, it is said to be 24 carats fine; if there is 22 parts of pure gold and 2 parts of alloy it is said to be 22 carats fine. The standard of American coin is nine tenths pure gold, and is worth $20.67. What is called the *new standard*, used for watch cases, etc., is 18 carats fine.

The term carat is also applied to a weight of $3\frac{1}{2}$ grains troy, used in weighing diamonds; it is divided into 4 parts called *grains;* 4 grains troy are thus equal to 5 grains diamond weight.

Apothecaries' Weight, used in Medical Prescriptions.

The pound and ounce of this weight are the same as the pound and ounce troy, but differently divided.

20 grains troy..1 scruple.	8 Drachms..1 ounce troy
3 scruples.......1 drachm.	12 ounces.....1 pound "

Druggists *buy* their goods by

Avoirdupois Weight.

By this weight all goods are sold except those named under troy weight.

$27\frac{11}{32}$ grains................1 drachm
16 drachms................1 ounce.
16 ounces................1 pound.

28 pounds	1 quarter.
4 quarters	1 hundred weight.
20 hundred weight	1 ton.

The grain avoirdupois, though never used, is the same as the grain in troy weight. 7,000 grains make the avoirdupois pound, and 5,760 grains the troy pound. Therefore, the troy pound is less than the avoirdupois pound in the proportion of 14 to 17, nearly; but the troy ounce is greater than the avoirdupois ounce in the proportion of 79 to 72, nearly. In times past it was the custom to allow 112 pounds for a hundred weight, but usage, as well as the laws of a majority of the States, at the present time call 100 pounds a hundred weight.

Apothecaries' Fluid Measure.

60 minims	1 fluid drachm.
8 fluid drachms	1 ounce (troy).
16 ounces (troy)	1 pint.
8 pints	1 gallon.

Measure of Capacity for all Liquors.

5 ounces, avoirdupois, of water make 1 gill.

4 gills	1 pint	= $34\frac{2}{3}$	cubic inches, nearly.
2 pints	1 quart	= $69\frac{1}{3}$	do.
4 quarts	1 gallon	= $277\frac{1}{4}$	do.

$31\frac{1}{2}$ gallons	1 barrel.
42 gallons	1 tierce.
63 gallons, or 2 bbls	1 hogshead.
2 hogsheads	1 pipe or butt.
2 pipes	1 ton.

The gallon must contain exactly 10 pounds avoirdupois of pure water at a temperature of 62°, the barometer being at 30 inches. It is the standard unit of measure of capacity for liquids and dry goods of every description, and is $\frac{1}{5}$ larger than the old wine measure, $\frac{1}{32}$ larger than the old dry measure, and $\frac{1}{60}$ less than the old ale measure. The wine gallon must contain 231 cubic inches.

Measure of Capacity for all Dry Goods.

4 gills	1 pint	=	$34\frac{2}{3}$ cub. in., nearly.
2 pints	1 quart	=	$69\frac{1}{3}$ cubic inches.
4 quarts	1 gallon	=	$277\frac{1}{4}$ cubic inches.
2 gallons	1 peck	=	$554\frac{1}{2}$ cubic inches.
4 pecks, or 8 gals.	1 bushel	=	$2150\frac{1}{2}$ cubic inches.
8 bushels	1 quarter	=	$10\frac{1}{4}$ cubic ft., nearly.

When selling the following articles a barrel weighs as here stated :

For rice, 600 lbs.; flour, 196 lbs.; powder, 25 lbs.; corn, as bought and sold in Kentucky, Tennessee, etc., 5 bushels of shelled corn; as bought and sold at New Orleans, a flour barrel full of ears; potatoes, as sold in New York, a barrel contains $2\frac{1}{4}$ bushels; pork, a barrel is 200 lbs., distinguished in quality by "clear," "mess," "prime;" a barrel of beef is the same weight.

The legal bushel of America is the old Winchester measure of 2,150.42 cubic inches. The imperial bushel of England is 2,218.142 cubic inches, so that 32 English bushels are about equal to 33 of ours.

Although we are all the time talking about the price of grain, etc., by the bushel, we sell by weight, as follows:

Wheat, beans, potatoes and clover seed, 60 lbs. to the bushel; corn, rye, flax seed and onions, 56 lbs.; corn on the cob, 70 lbs.; buckwheat, 52 lbs.; barley, 48 lbs.; hemp seed, 44 lbs.; timothy seed, 45 lbs.; castor beans, 46 lbs.; oats, 35 lbs.; bran, 20 lbs.; blue grass seed, 14 lbs.; salt, the real weight of coarse salt is 85 lbs.; dried apples, 24 lbs.; dried peaches, 33 lbs., according to some rules, but others are 22 lbs. per bushel, while in Indiana dried apples and peaches are sold by the heaping bushel; so are potatoes, turnips, onions, apples, etc., and in some sections oats are heaped. A bushel of corn in the ear is three heaped half bushels, or four even full.

In Tennessee a hundred ears of corn is sometimes counted as a bushel.

A hoop $18\frac{1}{2}$ inches diameter 8 inches deep holds a Winchester bushel. A box 12 inches square 7 and $7\frac{1}{32}$ deep will hold half a bushel. A heaping bushel is 2,815 cubic inches.

Cloth Measure.

$2\frac{1}{2}$ inches	1 nail.
4 nails	1 quarter of a yard.
4 quarters	1 yard.

Foreign Cloth Measure.

$2\frac{1}{2}$ quarters	1 ell Hamburg.
3 quarters	1 ell Flemish.
5 quarters	1 ell English.
6 quarters	1 ell French.

Measure of Length.

12 inches	1 foot.
3 feet	1 yard.
$5\frac{1}{2}$ yards	1 rod, pole or perch.
40 poles	1 furlong.
8 furlongs, or 1760 yds.	1 mile.
$69\frac{1}{15}$ miles	1 degree of a great circle of the earth.

By scientific persons and revenue officers the inch is divided into tenths, hundredths, etc. Among mechanics the inch is divided into eighths. The division of the inch into 12 parts, called lines, is not now in use.

A standard English mile, which is the measure that we use, is 5,280 feet in length, 1,760 yards, or 320 rods. A strip one rod wide and one mile long is two acres. By this it is easy to calculate the quantity of land taken up by roads, and also how much is wasted by fences.

Gunter's Chain.

USED FOR LAND MEASURE.

$7\frac{92}{100}$ inches	1 link.
100 links, or 66 feet, or 4 poles	1 chain.
10 chains long by 1 broad, or 10 sq. chains	1 acre.
80 chains	1 mile.

Surface Measure.

144	square inches	1 square foot.
9	square feet	1 square yard.
$30\frac{1}{4}$	square yards	1 square rod or perch.
40	square perches	1 rood.
4	roods	1 acre.
640	acres	1 square mile.

Measure 209 feet on each side and you have a square acre within an inch.

The following gives the comparative size in square yards of acres in different countries:

English acre, 4,840 square yards; Scotch, 6,150; Irish, 7,840; Hamburg, 11,545; Amsterdam, 9,722; Dantzic, 6,650; France (hectare), 11,960; Prussia (morgen), 3,053.

This difference should be borne in mind in reading of tne products per acre in different countries. Our land measure is that of England.

Government Land Measure.

A township—36 sections, each a mile square.

A section—640 acres.

A quarter section, half a mile square—160 acres.

An eighth section, half a mile long, north and south, and a quarter of a mile wide—80 acres.

A sixteenth section, a quarter of a mile square—40 acres.

The sections are all numbered 1 to 36, commencing at the northeast corner thus:

6	5	4	3	2	N W, N E, S W, S E
7	8	9	10	11	12
18	17	16*	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

The sections are divided into quarters, which are named by the cardinal points, as in section 1. The quarters are divided in the same way. The description of a forty acre lot would read: The south half of the west half of the southwest quarter of section 1 in township 24, north of range 7 west, or as the case might be; and sometimes will fall short and sometimes overrun the number of acres it is supposed to contain.

* School section.

Square Measure,

FOR CARPENTERS, MASONS, ETC.

144 square inches........1 square foot.
9 sq. ft., or 1,296 sq. in.1 square yard.
100 square feet..........1 sq. of flooring, roofing, &c.
$30\frac{1}{4}$ square yards........1 square rod.
36 square yards........1 rood of building.

Geographical or Nautical Measure.

6 feet.................1 fathom.
110 fathoms, or 660 feet..1 furlong.
$6075\frac{4}{5}$ feet...............1 nautical mile.
3 nautical miles.......1 league.
20 leag's, or 60 geo. miles.1 degree.
360 degrees.............The earth's circumference $=24,855\frac{1}{2}$ miles, nearly.

The nautical mile is $795\frac{4}{5}$ feet longer than the common mile.

Measure of Solidity.

1728 cubic inches.........1 cubic foot.
27 cubic feet...........1 cubic yard.
16 cubic feet...........1 cord ft., or ft. of wood.
8 cord ft., or 128 cub. ft.1 cord.
40 ft. of round, or 50 ft. of hewn timber, } 1 ton.
42 cubic feet...........1 ton of shipping.

Angular Measure, or Divisions of the Circle.

60 seconds.....................1 minute.
60 minutes.....................1 degree.
30 degrees.....................1 sign.
90 degrees.....................1 quadrant.
360 degrees.....................1 circumference.

Measure of Time.

60 seconds	1 minute.
60 minutes	1 hour.
24 hours	1 day.
7 days	1 week.
28 days	1 lunar month.
28, 29, 30 or 31 days	1 calen'r month.
12 calendar months	1 year.
365 days	1 com. year.
366 days	1 leap year.
365¼ days	1 Julian year.
365 d. 5 h. 48 m. 49 s	1 Solar year.
365 d. 6 h. 9 m. 12 s	1 Siderial year.

Ropes and Cables.

6 feet	1 fathom.
120 feet	1 cable length.

MISCELLANEOUS IMPORTANT FACTS ABOUT WEIGHTS AND MEASURES.

Board Measure.

Boards are sold by superficial measure at so much per foot of one inch or less in thickness. adding one fourth to the price for each quarter inch thickness over an inch.

Grain Measure in Bulk.

Multiply the width and length of the pile together, and that product by the height, and divide by 2,150, and you have the contents in bushels.

If you wish the contents of a pile of ears of corn, or roots, in heaped bushels, ascertain the cubic inches and divide by 2,818.

A Ton Weight.

In this country a ton is 2,000 pounds. In most places a ton of hay, etc., is 2,240 pounds, and in some places that foolish fashion still prevails of weighing all bulky articles sold by the ton by the "long weight," or tare of 12 lbs. per cwt.

A ton of round timber is 40 feet; of square timber, 54 cubic feet.

ANIMAL STRENGTH.

Men.

The mean effect of the power of a man, unaided by a machine, working to the best practicable advantage, is the raising of 70 lbs. 1 foot high in a second, for 10 hours in a day.

Two men, working at a windlass at right angles to each other, can raise 70 lbs. more easily than one man can 30 lbs.

The result of observation upon animal power furnishes the following as the maximum daily effect:

1. When the effect produced varied from $\frac{1}{3}$ to .2 of that which could be produced without velocity during a brief interval.

2. When the velocity varied from $\frac{1}{4}$ to $\frac{1}{6}$ for a man, and from .08 to .066 for a horse, of the velocity which they were capable for a brief interval, and not producing any effort.

3. When the duration of the daily work varied from $\frac{1}{2}$ to $\frac{1}{3}$ for a brief interval, during which the work could be constantly sustained without prejudice to the health of the man or the animals—the time not extending beyond 18 hours per day, however limited may be the daily task, so long as it represents a constant attendance in the shop.

By Mr. Field's experiments in 1838 the maximum power of a strong man, exerted for $2\frac{1}{2}$ minutes=18,000 lbs. raised one foot in a minute.

A man of ordinary strength exerts a force of 30 lbs. for 10 hours in a day, with a velocity of $2\frac{1}{2}$ feet in a second=4,500 raised one foot in a minute=.2 of the work of a horse.

A man can travel, without a load, on level ground, during $8\frac{1}{2}$ hours a day at the rate of 3.7 miles an hour, or $31\frac{1}{4}$ miles a day. He can carry 111 lbs. 11 miles in a day. Daily allowance of water for a man, one gallon for all purposes, and he requires from 220 to 240 cubic feet of air per hour.

A porter, going short distances and returning unloaded, can carry 135 lbs. 7 miles a day. He can transport in a wheelbarrow 150 lbs. 10 miles in a day.

The muscles of the human jaw exert a force of 534 lbs.

Mr. Buchanan ascertained that, in *working a pump, turning a winch, in ringing a bell* and *in rowing a boat* the effective power of a man is as the numbers 100, 167, 227 and 248.

A man *drawing a boat in a canal* can transport 110,000 lbs. for a distance of 7 miles, and produce 156 times the effect of a man weighing 154 lbs. and walking $31\frac{1}{4}$ miles in a day. He can also produce an effect upon a *tread-wheel* of 30 lbs., with a velocity of 2.3 feet in a second, for 7 hours in a day, and can *draw* or *push* on a horizontal plane 30 lbs. with a velocity of 2 feet in a second for 8 hours in a day. He can raise by a *single pulley* 38 lbs. with a velocity of .8 of a foot per second for 8 hours in a day, and he can pass over $12\frac{1}{2}$ times the space horizontally that he can vertically.

A foot soldier travels in 1 minute, in common time, 90 steps=70 yards.

A foot soldier travels in 1 minute, in quick time, 110 steps=86 yards.

A foot soldier travels in 1 minute, in double-quick time, 140 steps=110 yards.

He occupies in the ranks a front of 20 inches and and a depth of 13, without a knapsack; the interval between the ranks is 13 inches.

Average weight of men, 150 lbs. each.

Five men can stand in a space of 1 square yard.

TABLE OF THE EFFECTIVE POWER OF MEN FOR A SHORT PERIOD.

MANNER OF APPLICATION.	FORCE.	MANNER OF APPLICATION.	FORCE.
	Lbs.		Lbs.
Bench vice or chisel.......	72	Screwdriver, one hand....	84
Brace bit.....	16	Small screwdriver.........	14
Drawing knife or auger....	100	Thumb and fingers.......	14
Hand plane....	50	Thumb vice..............	45
Hand-saw...............	36	Windlass or pincers.......	60

Horses.

A horse can travel 400 yards at a walk in $4\frac{1}{2}$ minutes, at a trot in 2 minutes, and at a gallop in 1 minute. He occupies in the ranks a front of 40 inches and a depth of 10 feet; in a stall, from $3\frac{1}{2}$ to $4\frac{1}{2}$ feet front; and at a picket, 3 feet by 9; and his average weight=1,000 lbs.

A horse, carrying a soldier and his equipments (225 lbs.), can travel 25 miles in a day (8 hours).

A draught horse can draw 1,600 lbs. 23 miles a day, weight of carriage included.

The ordinary work of a horse may be stated at

22,500 lbs., raised 1 foot in a minute, for 8 hours a day.

In a horse-mill a horse moves at the rate of 3 feet in a second. The diameter of the track should not be less than 25 feet.

A horse-power in machinery is estimated at 33,000 lbs., raised 1 foot in a minute; but as a horse can exert that force but 6 hours in a day, one machinery horse-power is equivalent to that of $4\frac{1}{2}$ horses.

The expense of conveying goods at 3 miles per hour per horse teams being 1, the expense at $4\frac{1}{3}$ miles will be 1.33, and so on—the expense being doubled when the speed is $5\frac{1}{8}$ miles per hour

The strength of a horse is equivalent to that of 5 men.

The daily allowance of water for a horse should be 4 gallons.

TABLE OF THE AMOUNT OF LABOR A HORSE OF AVERAGE STRENGTH IS CAPABLE OF PERFORMING AT DIFFERENT VELOCITIES, ON CANALS RAILROADS AND TURNPIKES.

Force of Traction estimated at 83.3 *lbs.*

VELOCITY PER HOUR.	DURATION OF WORK.	USEFUL EFFECT FOR ONE DAY, DRAWN ONE MILE.		
		ON A CANAL.	ON A RAILR'D	ON A T'NPIKE.
MILES.	HOURS.	TONS.	TONS.	TONS.
2½	11.5	520	115	14
3	8.	243	92	12
4	4.5	102	72	9
5	2.9	52	57	7.2
6	2.	30	48	6
7	1.5	19	41	5.1
8	1.125	12.8	36	4.5
10	.75	6.6	28.8	3.6

The actual labor performed by horses is greater, but they are injured by it.

A horse in a mill can produce an effect of 106 lbs. at a velocity of 3 feet in a second for 8 hours in a day. A mule can produce, under a like velocity and time, an effect of 71 lbs., and an ass 37 lbs.

An ox, walking at a velocity of 2 feet in a second (1.34 miles per hour), will draw 154 lbs. for 8 hours in a day.

A horse requires a space of 7 feet by $2\frac{1}{2}$ for transportation in a vessel; and a beef requires $6\frac{1}{4}$ feet by 26 inches, without manger, and 2 feet additional length with one. 3 beeves or 15 sheep require the food of 2 horses.

TABLE SHOWING THE AMOUNT OF LABOR PRODUCED BY ANIMAL POWER UNDER DIFFERENT CIRCUMSTANCES.

MANNER OF APPLICATION.	Power.	Velocity per Second.	Weight raised. Foot per Minute.	Horses Power for the Period given.
10 HOURS PER DAY.	Lbs.	Feet.	Lbs.	No.
Man, throwing earth with a shovel, a height of 5 feet	6	1⅛	480	8.7
Man, wheeling a loaded barrow up an inclined plane, height one twelfth of length	132	⅝	4950	90.
Man, raising and pitching earth in a shovel to a horizontal distance of 13 feet	6	2¼	810	14.7
Man, pushing and drawing alternately in a vertical direction	13	2½	1950	35.5
Man, transporting weight upon a barrow, and returning unloaded	132	1	7920	144
Man, walking upon a level	143	5	42900	780
Horse, drawing a four-wheeled carriage at a walk	154	3	27720	504
Horse, with load upon his back, at a walk.	264	3¾	59400	1080
Horse, transporting a loaded wagon, and returning unloaded at a walk	1540	2	184800	3360
Horse, drawing a loaded wagon at a walk	1540	3¾	346500	6300
8 HOURS PER DAY.				
Man, ascending a slight elevation, unloaded	143	½	4290	62
Man, walking, and pushing or drawing in a horizontal direction	26	2	3120	45.2
Man, turning a crank	18	2½	2790	39
Man, upon a tread-mill	140	½	4200	60.9
Man, rowing	26	5	7800	113
Horse, upon a revolving platform, at a walk	100	3	18000	260.8
Ox, upon a revolving platform, at a walk	132	2	15840	229.5
Mule, upon a revolving platform, at a walk	66	3	11880	172.2
Ass, upon a revolving platform, at a walk	32	2¾	5280	76.5
7 HOURS PER DAY.				
Man, walking with a load upon his back	88	2½	13200	167.9
6 HOURS PER DAY.				
Man, transporting a weight upon his back, and returning unloaded	140	1¾	14700	160.5
Man, transporting a weight upon his back up a slight elevation, and returning unloaded	140	.2	1680	19
Man, raising a weight by the hands	44	½	1320	14.4
4½ HOURS PER DAY.				
Horse, upon a revolving platform at a trot.	66	6¾	26730	218.7
Horse, drawing an unloaded four-wheeled carriage at a trot	97	7¼	43195	353.5
Horse, drawing a loaded four-wheeled carriage at a trot	770	7¼	334950	2741

How many men are required upon a tread-mill, 20 feet in diameter, in order to raise a weight of 900 lbs., the crank being 9 inches in length?

The weight of the wheel and its load is estimated at 5000 lbs., and the friction at .015=at 75 lbs. The labor of a man upon such a mill is estimated at 25 lbs. Length of crank=.75 feet.

Then $900 \times .75 + 5000 \times .015 = 750$ *lbs., the resistance of the wheel;* and $\frac{750}{20 \div 2} = 75$ *lbs., the power required at the circumference of the wheel.*

Therefore, $75 \div 25 = 3$ *men.*

The draught of man and animals by traces is as follows:

Man....150 lbs. Horse.....600 lbs. Mule.....500 lbs. Ass.....360 lbs.

A man rowing a boat 1 mile in 7 minutes performs the labor, while rowing, of 6 fully worked laborers at ordinary occupations of 10 hours.

TABLE SHOWING THE EFFECTS OF A TRACTION OF 100 LBS. AT DIFFERENT VELOCITIES ON CANALS.

Velocity per Hour.	Velocity per Second.	Mass Moved.	Useful Effect.	Velocity per Hour.	Velocity per Second.	Mass Moved.	Useful Effect.
Miles.	Feet.	Lbs.	Lbs.	Miles.	Feet.	Lbs.	Lbs.
2½	3.66	55500	39408	6	8.8	9635	6840
3	4.4	38542	27361	7	10.26	7080	5026
3½	5.13	28316	20100	8	11.73	5420	3848
4	5.86	21680	15390	9	13.2	4282	3040
5	7.33	13875	9850	10	14.66	3468	2462

The load carried, added to the weight of the vessel which contains it, forms the total mass moved, and the useful effect is the load.

The force of traction on a railroad or turnpike is constant, but the mechanical power necessary to move the carriage increases with the velocity. On a canal the force of traction varies as the square of the velocity.

Labor upon Embankments.

ELLWOOD MORRIS.

Single Horse and Cart.—A horse with a loaded dirt cart, employed in excavation and embankment, will make 100 lineal feet of trip, or 200 feet in distance, per minute, while moving. The time lost in loading, dumping, awaiting, etc.,=4 minutes per load.

A medium laborer will load with a cart in 10 hours, of the following earths, measured in the bank:

Gravelly earth, 10; *loam*, 12; and *sandy earth*, 14 cubic yards.

Earth from a natural excavation occupies $\frac{1}{9}$ more space than when transported to an embankment.

Carts are loaded as follows: *Descending hauling*, $\frac{1}{8}$ of a cubic yard in bank; *level hauling*, $\frac{2}{7}$ of a cubic yard in bank; *ascending hauling*, $\frac{1}{4}$ of a cubic yard in bank.

Loosening, etc.—In *loam*, a three-horse plow will loosen from 250 to 800 cubic yards per day of 10 hours.

The cost of loosening earth to be loaded will be from 1 to 8 cents per cubic yard when wages are 105 cents per day.

The cost of trimming and bossing is about 2 cents per cubic yard.

Scooping.—A scoop load will measure $\frac{1}{10}$ of a cubic yard measured in excavation.

The time lost in loading, unloading and returning, per load, is $1\frac{1}{3}$ minutes.

The time lost for every 70 feet of distance from excavation to bank, and returning, is 1 minute.

In *double scooping* the time lost in loading, returning, etc., will be 1 minute, and in *single scooping* it will be $1\frac{1}{3}$ minutes.

VOLUMES OF EXCAVATION AND EMBANKMENT.

The volume of earth in embankment is less than in excavation, as the *compression* of earth in an embankment is in excess of the *expansion* of its volume in a natural state, the proportion being as follows: Sand, $\frac{1}{7}$; clay, $\frac{1}{9}$; gravel, $\frac{1}{4}$.

The volume of rock in bank exceeds that in excavation in the proportion of 3 to 2.

Measurement and Computation of the Tonnage of Vessels under the Act of Congress of 6th of May, 1864.

Measurements are expressed in feet and decimals of a foot, and tonnage in tons and hundredths of a ton.

The "tonnage length" is the length along the middle line of the vessel upon the *under side* of the tonnage deck plank, but, for convenience, is measured upon the *top* of the deck, and is the length between these extremities, which is divided into a number of parts, according to the classification made under the law.

The depths are perpendicular and the breadths horizontal; the upper breadth, which, in every case, passes through the top of the tonnage depth, being at a distance below the deck at its middle line equal to one third of the spring of the beam at that point, and thus passing *through* the deck upon each side; and the lower breadth, which is at the bottom of the tonnage depth, being at a distance above the upper side of the floor timber at the inside of the limber strake, equal to the average thickness of the ceiling, and thus passing *through* the keelson.

The "spring of the beam" is the perpendicular distance from the crown of the tonnage deck at the centre to a line stretched from end to end of the beam, and must be ascertained at each point where it is to be used in the measurement.

The register of every vessel expresses her length and breadth, together with her depth, and the height under the third or spar deck, and is ascertained in the following manner: The tonnage deck, in vessels having three or more decks to the hull, is the second deck from below; in all other cases the upper deck of the hull is the tonnage deck. The length from the fore part of the outer planking upon the side of the stem to the after part of the main stern post of screw steamers, and to the after part of the rudder post of all other vessels, measured upon the top of the tonnage deck, is accounted the vessel's length. The breadth of the broadest part upon the outside of the vessel is accounted the vessel's breadth of beam. A measure from the under side of the tonnage deck plank amidships to the ceiling of the hold (average thickness) is accounted the depth of the hold. If the vessel has a third deck then the height from the top of the tonnage deck plank to the under side of the upper deck plank is accounted as the height under the spar deck.

The register tonnage of a vessel is her internal cubical capacity in tons of 100 cubic feet each, to be ascertained as follows: From the inside of the inner plank (average thickness) at the side of the stem to the inside of the plank upon the stern timbers (average thickness), deducting from this length what is due to the rake of the bow in the thickness of the deck, and what is due to the rake of the stern timber in the thickness of the deck, and also what is due to the rake of the stern timber in one third of the spring of the beam.

CLASSES.

CLASS 1. Vessels of which the tonnage length is 50 feet or under.
" 2. Over 50 feet, and not exceeding 100 feet in length.
" 3. Over 100 feet, and not exceeding 150 feet in length.
" 4. Over 150 feet, and not exceeding 200 feet in length.
" 5. Over 200 feet, and not exceeding 250 feet in length.
" 6. Over 250 feet in length.

If there is a break, a poop, or any other permanent closed in space upon the upper decks, or upon the spar deck, available for cargo or stores, or for the berthing or accommodation of passengers or crew, the tonnage of such space is computed.

If a vessel has a third deck, or a spar deck, the tonnage of the space between it and the tonnage deck is computed.

In computing the tonnage of open vessels the upper edge of the upper strake is to form the boundary line of measurement, and the depth should be taken from an athwart-ship line, extending from the upper edge of said strake at each division of the length.

The register of a vessel expresses the number of decks, the tonnage under the tonnage deck, that of the between decks above the tonnage deck, also that of the poop or other enclosed space above the deck, each separately. In every registered United States vessel the number denoting the total registered tonnage must be deeply carved, or otherwise permanently marked upon her main beam, and shall be so continued; and if at any time it cease to be so continued, such vessel shall no longer be recognized as a registered United States vessel.

RECAPITULATION OF MEASUREMENTS.

Registered Length.—Length at the middle of the second deck from below in vessels of two or more decks, and in all other vessels of the upper deck, measured from the fore part of the outer planking upon the side of the stem to the after part of the main stern post of single screw propeller steamers, and to the after part of the rudder post of other vessels, measured upon the top of the tonnage deck.

Tonnage Length.—Length at upper side of tonnage deck beams from inside of the inboard plank, at its average thickness at the side of the stem, to the inside of the plank upon the stern timbers, at its average thickness, deducting from this length that which is due to the rake of the bow in the thickness of the deck, and of the stern timber in the thickness of the deck, and one third the spring of the beam.

Breadth of Beam.—At the broadest part of the outside of the vessel.

Depth of Hold.—Height measured from the under side of tonnage deck plank amidships, from a point at a distance of one third the spring of the beam to the ceiling of the hold at its average thickness.

Height under Spar Deck.—The mean height from top of tonnage deck plank to the under side of the upper deck plank.

Open Vessels.—The upper edge of the upper strake is to be the boundary line of measurement of length, and the depth is to be measured from a line running athwart-ships from the upper edge of the upper strake at each division of the length.

By an Act of Congress of 28th of February, 1865, the preceding rule of admeasurement was amended as follows: No part of any ship or vessel shall be admeasured or registered for tonnage that is used for cabins or state-rooms, and constructed entirely above the first deck, which is not a deck to the hull.

CARPENTER'S MEASUREMENT.—FOR A SINGLE DECK VESSEL.

Rule.—Multiply the length of keel, the breadth of beam, and the depth of hold together, and divide by 95.

FOR A DOUBLE DECK VESSEL.

Rule.—Multiply as above, taking half the breadth of beam for the depth of the hold, and divide by 95.

BRITISH MEASUREMENT.

Divide the length of the upper deck between the after part of the stem and the fore part of the stern post into 6 equal parts, and note the foremost, middle and aftermost points of division. Measure the depths at these three points in feet and tenths of a foot, also the depths from the under side of the upper deck to the ceiling of the limber strake, or, in case of a break in the upper deck, from a line stretched in continuation of the deck. For the breadths divide each depth into 5 equal parts, and measure the inside breadths at the following points, viz., at .2 and .8 from the upper deck of the foremost and aftermost depths, and at .4 and .8 from the upper deck of the midship depth. Take the length at half the midship depth from the after part of the stem to the fore part of the stern post. Then, to twice the midship depth add the foremost and aftermost depths for the sum of the depths, and add together the foremost upper and lower breadths—3 times the upper breadths with the lower breadth at the midship, and the upper and twice the lower breadth at the after division for the sum of the breadths.

Multiply together the sum of the depths, the sum of the breadths and length, and divide the product by 3500, which will give the number of tons, or register.

If the vessel has a poop or half deck, or a break in the upper deck, measure the inside mean length, breadth and height of such part thereof as may be included within the bulkhead; multiply these three measurements together and divide the product by 92.4. The quotient will be the number of tons to be added to the result as above ascertained.

For Open Vessels.—The depths are to be taken from the upper edge of the upper strake.

For Steam Vessels.—The tonnage due to the engine room is deducted from the total tonnage computed by the above rule.

To determine this measure the inside length of the engine room from the foremost to the aftermost bulkhead, then multiply this length by the midship depth of the vessel, and the product by the inside midship breadth at .4 of the depth from the deck, and divide the final product by 92.4.

TABLE OF MULTIPLES.

For the practical convenience of those who have occasion to refer to mensuration, we have arranged the following useful table of multiples. It covers the whole ground of practical geometry, and should be studied carefully by those who wish to be skilled in this beautiful branch of mathematics.

Diameter of a circle$\times$3.1416=circumference.

Radius of a circle$\times$6.283185=circumference.

Square of the radius of a circle$\times$3.1416=area.

Square of the diameter of a circle$\times$0.7854=area.

Square of the circumference of a circle$\times$0.07958=area.

Half the circumference of a circle$\times$by half its diameter=area.

Circumference of a circle$\times$0.159155=radius.

Square root of the area of a circle$\times$0.56419=radius.

Circumference of a circle$\times$0.31831=diameter.

Square root of the area of a circle$\times$112838=diameter.

Diameter of a circle$\times$0.86=side of inscribed equilateral triangle.

Diameter of a circle$\times$0.7071=side of an inscribed square.

Circumference of a circle$\times$0.225=side of an inscribed square.

Circumference of a circle$\times$0.282=side of an equal square.

Diameter of a circle$\times$0.8862=side of an equal square.

Base of a triangle$\times$by ½ the altitude=area.

Multiplying both diameters and .7854 together =area of an ellipse.

Surface of a sphere × by $\frac{1}{6}$ of its diameter = solidity.

Circumference of a sphere × by its diameter = surface.

Square of the diameter of a sphere × 3.1416 = surface.

Square of the circumference of a sphere × 0.3183 = surface.

Cube of the diameter of a sphere × 0.5236 = solidity.

Cube of the radius of a sphere × 4.1888 = solidity

Cube of the circumference of a sphere × 0.016887 = solidity.

Square root of the surface of a sphere × 0.56419 = diameter.

Square root of the surface of a sphere × 1.772454 = circumference.

Cube root of the solidity of a sphere × 1.2407 = diameter.

Cube root of the solidity of a sphere × 3.8978 = circumference.

Radius of a sphere × 1.1547 = side of inscribed cube.

Square root of ($\frac{1}{3}$ of the square of) the diameter of a sphere = side of inscribed cube.

Area of its base × by $\frac{1}{3}$ of its altitude = solidity of a cone or pyramid, whether round, square or triangular.

Area of one of its sides × 6 = surface of a cube.

Altitude of rapezoid × $\frac{1}{2}$ the sum of its parallel sides = area.

WOOD, TIMBER, etc.

Selection of Standing Trees.—*Wood* grown in a moist soil is lighter, and decays sooner than that grown in dry, sandy soil.

The best timber is that grown in a dark soil intermixed with gravel. Poplar, cypress, willow, and all others which grow best in a wet soil, are exceptions.

The hardest and densest woods, and the least subject to decay, grow in warm climates, but they are more liable to split and warp in seasoning.

Trees grown upon plains or in the centre of forests are less dense than those from the edge of a forest, from the side of a hill, or from open ground.

Trees (in the United States) should be selected in the latter part of July or first part of August—for at this season the leaves of the sound, healthy trees are fresh and green, while those of the unsound are beginning to turn yellow. A sound, healthy tree is recognized by its top branches being well leaved, the bark even and of a uniform color. A rounded top, few leaves, some of them turned yellow, a rougher bark than common, covered with parasitic plants, and with streaks or spots upon it, indicate a tree upon the decline. The decay of branches and the separation of bark from the wood, are infallible indications that the wood is impaired.

Felling Timber.—The most suitable time for felling timber is in midwinter and in midsummer. Recent experiments indicate the latter season, and in the month of July.

A tree should be allowed to attain full maturity before being felled. Oak matures at 75 to 100 years and upward, according to circumstances. The age and rate of growth of a tree are indicated by the number and width of the rings of annual increase which are exhibited in a cross section.

A tree should be cut as near the ground as practicable, as the lower part furnishes the best timber.

Dressing Timber.—As soon as a tree is felled it should be stripped of its bark, raised from the ground, the sap wood taken off and the timber reduced to its required dimensions.

Inspection of Timber.—The quality of wood is in some degree indicated by its color, which should be nearly uniform in the heart, a little deeper toward the centre, and free from sudden transitions of color. White spots indicate decay. The sap wood is known by its white color; it is next to the bark, and very soon rots.

Defects of Timber.—*Wind-shakes* are circular cracks, separating the concentric layers of wood from each other. It is a serious defect.

Splits, checks and cracks, extending toward the centre, if deep and strongly marked, render the timber unfit for use, unless the purpose for which it is intended will admit of its being split through them.

Brushwood is generally consequent upon the decline of the tree from age. The wood is porous, of a reddish color, and breaks short without splinters.

Belted Timber is that which has been killed before being felled, or which has died from other causes. It is objectionable.

Knotty Timber is that containing many knots, though sound; usually of stunted growth.

Twisted Wood is when the grain of it winds spirally; it is unfit for long pieces.

Dry rot.—This is indicated by yellow stains. Elm and beech are soon affected if left with the bark on.

Large or decayed knots injuriously affect the strength of timber.

Seasoning and Preserving Timber.

Timber freshly cut contains about 37 to 48 per cent. of liquids. By exposure to the air in seasoning one year it loses from 17 to 25 per cent.; and, when seasoned, it yet retains from 10 to 15 per cent.

Timber of large dimensions is improved and rendered less liable to warp and crack in being seasoned by immersion in water for some weeks.

For the purpose of seasoning timber should be piled under shelter and be kept dry. It should have a free circulation of air about it, without being exposed to strong currents. The bottom pieces should be placed upon skids, which should be free from decay, raised not less than two feet from the ground; a space of an inch should intervene between the pieces of the same horizontal layers, and slats or piling strips placed between each layer, one near each end of the pile and others at short distances, in order to keep the timber from winding. These strips should be one over the other—and, in large piles, should not be less than one inch thick. Light timber may be piled in the upper portion of the shelter, heavy timber upon the ground floor. Each pile should contain but one description of timber. The piles should be at least two and a half feet apart.

Timber should be repiled at intervals, and all pieces indicating decay should be removed, to prevent their affecting those which are still sound.

Timber houses are best provided with blinds, which keep out rain and snow, but which can be turned to admit air in fine weather; and they should be kept entirely free from any pieces of decayed wood.

The gradual mode of seasoning is the most favorable to the strength and durability of timber; but various methods have been proposed for hastening the process. For this purpose *steaming* timber has been applied with success, and the results of experiments of various processes of saturating timber with a solution of corrosive sublimate and antiseptic fluids are very satisfactory. This process hardens and seasons wood, at the same time that it secures it from dry rot and from the attacks of worms. *Kiln drying* is serviceable only for boards and pieces of small dimensions, and is apt to cause cracks and to impair the strength of wood, unless performed very slowly. Charring or painting is highly injurious to any but seasoned timber, as it effectually prevents the drying of the inner part of the wood, in consequence of which fermentation and decay soon take place.

Timber piled in badly ventilated sheds is apt to be attacked with the common rot. The first outward indications are yellow spots upon the ends of the pieces, and a yellowish dust in the checks and cracks, particularly where the pieces rest upon the piling strips.

Timber requires from two to eight years to be seasoned thoroughly, according to its dimensions. It should be worked as soon as it is thoroughly dry, for it deteriorates after that time.

Oak timber loses one fifth of its weight in seasoning, and about one third of its weight in becoming perfectly dry. Seasoning is the extraction or dissipation of the vegetable juices and moisture, or the solidification of the albumen. When wood is exposed to currents of air at a high temperature the moisture evaporates too rapidly and the wood cracks, and when the temperature is high and sap remains it ferments, and dry rot ensues.

Timber is subject to *common rot* or *dry rot*—the former occasioned by alternate exposure to moisture and dryness. The progress of this decay is from the exterior; hence, the covering of the surface with paint, tar, etc., is a preservative.

Painting and charring *green* timber hastens its decay.

Dry or sap rot is inherent in timber, and it is occasioned by the putrefaction of the vegetable albumen. Sap wood contains a large proportion of fermentable elements. Insects attack wood for the sugar or gum contained in it, and fungi subsist upon the albumen of wood ; hence, to arrest dry rot the albumen must be either extracted or solidified.

In the seasoning of timber naturally there is required a period of from two to four years. Immersion in water facilitates seasoning by solving the sap.

The most effective method of preserving timber is that of expelling or exhausting its fluids, solidifying its albumen and introducing an antiseptic liquid.

The strength of impregnated timber is not reduced, and its *resilience* is improved.

In desiccating timber, by expelling its fluids by heat and air, its strength is increased fully 15 per cent.

In coating unseasoned timber with creosote, tar, etc., it is also preserved from the attack of worms. Jarrow wood, from Australia, is not subjected to their attack.

The condition of timber, as to its soundness or decay, is readily recognized when struck a quick blow.

Timber that has been for a long time immersed in water, when brought into the air and dried, becomes brashy and useless.

When trees are barked in the spring they should not be felled until the foliage is dead.

Timber cannot be seasoned by either smoking or charring; but when it is to be used in locations where it is exposed to worms or to produce fungi, it is proper to smoke or char it.

Timber may be partially seasoned by being boiled or steamed

Impregnation of Wood.

The several processes are as follows:

Kyan, 1832.—Saturated with corrosive sublimate. Solution, 1 lb. of chloride of mercury to 4 gallons of water

Burnett, 1838.—Impregnation with chloride of zinc, by submitting the wood endwise to a pressure of 150 lbs. per square inch. Solution, 1 lb. of the chloride to 10 gallons of water.

Boucheri.—Impregnation, by submitting the wood endwise to a pressure of about 15 lbs. per square inch. Solution, 1 lb. of sulphate of copper to 12½ gallons of water.

Bethel.—Impregnation, by submitting the wood endwise to a pressure of 150 to 200 lbs. per square inch, with oil of creosote mixed with bituminous matter.

Louis S. Robbins, 1865.—Aqueous vapor dissipated by the wood being heated in a chamber, the albumen solidified then submitted to the vapor of coal tar, resin, or bituminous oils, which, being at a temperature not less than 325°, readily takes the place of the vapor expelled by a temperature of 212°.

Fluids will pass with the grain of wood with great facility, but will not enter it except to a very limited extent when applied externally.

Absorption of Preserving Solution by different Woods for a period of seven days.

AVERAGE POUNDS PER CUBIC FOOT.

Black Oak	3,6	Hemlock	2.6	Rock Oak	3.9
Chestnut	3.	Red Oak	3.9	White Oak	3.1

PROPORTION OF WATER IN VARIOUS WOODS.

Alder (*Betula alnus*)	41.6	Pine (*Pinus Sylvestris L.*)	39.7
Ash (*Fraxinus excelsior*)	28.7	Red Beech (*Fagus sylvatica*)	39.7
Birch (*Betula alba*)	30.8	Red Pine (*Pinus picea dur*)	45.2
Elm (*Ulmus campestris*)	44.5	Sycamore (*Acer pseudo-platanus*)	27.
Horse Chestnut (*Æsculus hippocast*)	38.2	White Oak (*Quercus alba*)	36.2
Larch (*Pinus larix*)	48.6	White Pine (*Pinus abies dur*)	37.1
Mountain Ash (*Sorbus aucuparia*)	28.3	White Poplar (*Populus alba*)	50.6
Oak (*Quercus robur*)	34.7	Willow (*Salix caprea*)	26.

COMPARATIVE RESILIENCE OF TIMBER.

Ash	1.00	Chestnut	.73	Larch	.84	Spruce	.64
Beech	.86	Elm	.54	Oak	.63	Teak	.59
Cedar	.66	Fir	.4	Pitch Pine	.57	Yellow Pine	.64

Weight and Strength of Oak and Yellow Pine.

WEIGHT OF A CUBIC FOOT.

AGE.	White Oak, Va.		Yellow Pine, Va.		Live Oak.
	Round.	Square.	Round.	Square.	
Green	64.7	67.7	47.8	39.2	78.7
One Year	53.6	53.5	39.8	34.2	—
Two Years	46.	49.9	34.3	33.5	66.7

In England timber sawed into boards is classed as follows:

6½ to 7 inches in width, *Battens;* 8½ to 10 inches, *Deals;* and 11 to 12 inches, *Planks.*

In a perfectly dry atmosphere the durability of woods is almost unlimited. Rafters of roofs are known to have existed 1,000 years, and piles submerged in fresh water have been found perfectly sound 800 years from the period of their being driven.

Distillation.—From a single cord of pitch pine, distilled by chemical apparatus, the following substances, and in the quantities stated, have been obtained:

Charcoal	50 bush.	Pyroligneous Acid	100 galls.
Illuminating Gas...about	1000 cu. ft.	Spirits of Turpentine	20 "
Illuminating Oil and Tar	50 galls.	Tar	1 bbl.
Pitch or Resin	1½ bbls.	Wood Spirit	5 galls.

DECREASE IN DIMENSIONS OF TIMBER BY SEASONING.

Woods.	Ins.	Ins.	Woods.	Ins.	Ins.
Cedar, Canada	14	to 13¼	Pitch Pine, South	18⅜	to 18¼
Elm	11	to 10¾	Spruce	8½	to 8⅜
Oak, English	12	to 11⅝	White Pine, American	12	to 11⅞
Pitch Pine, North	10x10	to 9¾x9¾	Yellow Pine, North	18	to 17⅞

The weight of a beam of English oak, when wet, was reduced by seasoning from 972.25 to 630.5 lbs.

IRON.

The foreign substances which iron contains modify its essential properties. Carbon adds to its hardness, but destroys some of its qualities, and produces cast iron or steel according to the proportion it contains. Sulphur renders it fusible, difficult to weld, and brittle when heated, or "hot short." Phosphorus renders it "cold short," but may be present in the proportion of $\frac{2}{1000}$ to $\frac{3}{1000}$ without affecting injuriously its tenacity. Antimony, arsenic and copper have the same effect as sulphur, the last in a greater degree.

Cast Iron.

The process of making cast iron depends much upon the description of fuel used—whether charcoal, coke, bituminous or anthracite coals. A larger yield from the same furnace, and a great economy in fuel are effected by the use of a *hot blast*. The greater heat thus produced causes the iron to combine with a larger percentage of foreign substances.

Cast iron, for purposes requiring great strength, should be smelted with a *cold blast*. Pig iron, according to the proportion of carbon which it contains, is divided into foundry iron and forge iron—the latter adapted only to conversion into malleable iron, while the former, containing the largest proportion of carbon, can be used either for castings or bars.

There are many varieties of cast iron, differing by almost insensible shades; the two principal divisions are *gray* and *white*, so termed from the color of their fracture. Their properties are very different.

Gray iron is softer and less brittle than white iron; it is in a slight degree malleable and flexible, and is not sonorous; it can be easily drilled or turned in a lathe, and does not resist the file. It has a brilliant fracture, of a gray (or sometimes a bluish-gray) color. The color is lighter as the grain becomes closer, and its hardness increases at the same time. It melts at a lower heat than white iron, and preserves its fluidity longer. The color of the fluid metal is red, and deeper in proportion as the heat is lower; it does not adhere to the ladle; it fills the moulds well, contracts less, and contains fewer cavities than white iron; the edges of its castings are sharp and the surfaces smooth and convex. A medium sized grain, bright gray color, fracture sharp to the touch, and a close, compact texture, indicate a good quality of iron. A grain either very large or very small, a dull, earthy aspect, loose texture, dissimilar crystals mixed together, indicate an inferior quality.

Gray iron is used for machinery and ordnance purposes, where the pieces are to be bored or fitted. Its tenacity and specific gravity are diminished by annealing. Its mean specific gravity is 7.2.

White iron is very brittle and sonorous; it resists the file and the chisel, and is susceptible of high polish. The surface of its castings is concave; the fracture presents a silvery appearance, generally fine grained and compact, sometimes radiating or lamellar. When melted it is white and throws off a great number of sparks, and its qualities are the reverse of those of gray iron; it is, therefore, unsuitable for machinery purposes. Its tenacity is increased and its specific gravity diminished by annealing. Its mean specific gravity is 7.5.

Mottled iron is a mixture of white and gray; it has a spotted appearance; it flows well and with few sparks; its castings have a plain surface with edges slightly rounded ; it is suitable for shot, shell, etc.

A fine mottled iron is the only kind suitable for castings which require great strength, such as beam centres, cylinders and cannon. The kind of mottle will depend much upon the size of the casting.

Besides these general divisions the different varieties of pig iron are more particularly distinguished by numbers, according to their relative hardness.

No. 1 is the softest iron, possessing in the highest degree the qualities belonging to gray iron; it has not much strength, but on account of its fluidity when melted, and of its mixing advantageously with old or scrap iron, and with the harder kinds of cast iron, it is of great use to the founder, and commands the highest price.

No. 2 is harder, closer grained and stronger than No. 1; it has a gray color and considerable lustre. It is the character of iron most suitable for shot and shell.

No. 3 is still harder than No. 2; its color is gray, but inclining to white; it has considerable strength, but it is principally used for mixing with other kinds of iron.

No. 4 is bright iron; No. 5 mottled, and No. 6 white, which is unfit for general use by itself.

The quality of these various descriptions depend upon the proportion of carbon, and upon the state in which it exists in the metal. In the darker kinds of iron, where the proportion is sometimes 7 per cent., it exists partly in the state of graphite or plumbago, which makes the iron soft. In white iron the carbon is thoroughly combined with the metal, as in steel.

Cast iron frequently retains a portion of foreign ingredients from the ore, such as earths or oxides of other metals, and sometimes sulphur and phosphorus, which are all injurious to its quality. Sulphur hardens the iron, and, unless in a very small proportion, destroys its tenacity.

These foreign substances, and also a portion of the carbon, are separated by melting the iron in contact with air, and soft iron is thus rendered harder and stronger. The effect of remelting varies with the nature of the iron and the character of the ore from which it has been extracted—that from the hard ores, such as the magnetic oxides, undergoes less alteration than that from the hematites, the latter being sometimes changed from No. 1 to white by a single remelting in an air furnace.

The color and texture of cast iron depend greatly upon the volume of the casting and the rapidity of its cooling—a small casting, which cools quickly, is almost always white, and the surface of large castings partakes more of the qualities of white metal than the interior.

All cast iron expands at the moment of becoming solid and contracts in cooling. Gray iron expands more and contracts less than other iron. The contraction is about $\frac{1}{100}$ for gray and strongly mottled iron, or $\frac{1}{8}$ of an inch per foot.

Remelting iron improves its tenacity; thus, a mean of 14 cases for two fusions gave, for 1st fusion, a tenacity of 29,284 lbs.; for 2d fusion, 33,790 lbs. And two cases, for 1st fusion, 15.129 lbs.; for 2d fusion, 35,786 lbs.

Wrought Iron.

Wrought iron is made from the pig iron in a bloomery fire or in a puddling furnace—generally in the latter. The process consists in melting it and keeping it exposed to a great heat, constantly stirring the mass, bringing every part of it under the action of the flame until it loses its remaining carbon, when it becomes malleable iron. When, however, it is desired to obtain iron of the best quality, the pig iron should be refined.

Refining.—This operation deprives the iron of a considerable portion of its carbon; it is effected in a blast furnace, where the iron is melted by means of charcoal or coke, and exposed for some time to the action of a great heat; the metal is then run into a cast iron mould, by which it is formed into a large broad plate. As soon as the surface of the plate is chilled cold water is poured on to render it brittle.

The bloomery resembles a large forge fire, where charcoal and a strong blast are used; and the refined metal or the pig iron, after being broken into pieces of the proper size, is placed before the blast, directly in contact with charcoal; as the metal fuses it falls into a cavity left for that purpose below the blast, where the bloomer works it into the shape of a ball, which he places again before the blast, with fresh charcoal; this operation is generally again repeated, when the ball is ready for the shingler.

The puddling furnace is a reverberatory furnace, where the flame of bituminous coal is brought to act directly upon the metal. The metal is first melted, the puddler then stirs it, exposing each portion in turn to the action of the flame, and continues this as long as he is able to work it. When it has lost its fluidity he forms it into balls weighing from 80 to 100 pounds, which are next passed to the shingler.

Shingling is performed in a strong squeezer, or under the trip-hammer. Its object is to press out as perfectly as practicable the liquid cinder which the ball still contains; it also forms the ball into shape for the puddle rolls. A heavy hammer, weighing from six to seven tons, effects this object most thoroughly, but not so cheaply as the squeezer. The ball receives from fifteen to twenty blows of a hammer, being turned from time to time, as required; it is now termed a bloom, and is ready to be rolled or hammered; or the ball is passed once through the squeezer and is still hot enough to be passed through the puddle rolls.

Puddle Rolls.—By passing the bloom through different grooves in these rolls it is reduced to a rough bar, from three to four feet in length—its name conveying an idea of its condition, which is rough and imperfect.

Piling.—To prepare rough bars for this operation they are cut by a pair of shears into such lengths as are best adapted to the size of the finished bar required; the sheared bars are then piled one over the other, according to the volume required, when the pile is ready for balling.

Balling.—This operation is performed in the balling furnace, which is similar to the puddling furnace, except that its bottom or hearth is made up, from time to time, with sand; it is used to give a welding heat to the piles to prepare them for rolling.

Finishing Rolls.—The balls are passed successively between rollers of various forms and dimensions, according to the shape of the finished bar required.

The quality of the iron depends upon the description of pig iron used, the skill of the puddler, and the absence of deleterious substances in the furnace.

The strongest cast irons do not produce the strongest malleable iron.

For many purposes—such as sheets for tinning, best boiler plates, and bars for converting into steel—charcoal iron is used exclusively; and, generally, this kind of iron is to be relied upon, for strength and toughness, with greater confidence than any other, though iron of superior quality is made from pigs made with other fuel and with a hot blast. Iron for gun barrels has been lately made from anthracite hot blast pigs.

Iron is improved in quality by judicious working, reheating it, and hammering or rolling. Other things being equal, the best iron is that which has been wrought the most.

STEEL.

Steel is a compound of iron and carbon, in which the proportion of the latter is from 1 to 5 per cent., and even less in some kinds. Steel is distinguished from iron by its fine grain, and by the action of diluted nitric acid, which leaves a black spot upon steel, and upon iron a spot which is lighter colored, in proportion to the carbon it contains.

There are many varieties of steel, the principal of which are :

Natural Steel, obtained by reducing rich and pure descriptions of iron ore with charcoal, and refining the cast iron so as to deprive it of a sufficient portion of carbon to bring it to a malleable state. It is used for files and other tools.

Indian Steel, termed *wootz*, is said to be a natural steel, containing a small portion of other metals.

Blistered Steel, or steel of cementation, is prepared by the direct combination of iron and carbon. For this purpose the iron, in bars, is put in layers, alternating with powdered charcoal, in a close furnace, and exposed for seven or eight days to a heat of about 9000°, and then put to cool for a like period. The bars on being taken out are covered with blisters, have acquired a brittle quality, and exhibit in the fracture a uniform crystalline appearance. The degree of carbonization is varied according to the purposes for which the steel is intended, and the best qualities of iron (Russian and Swedish) are used for the finest kinds of steel.

Tilted Steel is made from blistered steel moderately heated, and subjected to the action of a tilt hammer, by which means its tenacity and density are increased.

Shear Steel is made from blistered or natural steel, refined by piling thin bars into fagots, which are brought to a welding heat in a reverberatory furnace, and hammered or rolled again into bars. This operation is repeated several times to produce the finest kinds of shear steel, which are distinguished by the names of "half shear," "single shear" and "double shear;" or steel of one, two or three *marks*, etc., according to the number of times it has been piled.

Cast Steel is made by breaking blistered steel into small pieces and melting it in close crucibles, from which it is poured into iron moulds; the ingot is then reduced to a bar by hammering or rolling. Cast steel is the best kind of steel, and best adapted for most purposes. It is known by a very fine, even and close grain, and a silvery, homogeneous fracture; it is very brittle and acquires extreme hardness, but is difficult to weld without the use of a flux.. The other kinds of steel have a similar appearance to cast steel, but the grain is coarser and less homogeneous; they are softer and less brittle, and weld more readily. A fibrous or lamellar appearance in the fracture indicates an imperfect steel. A material of great toughness and elasticity, as well as hardness, is made by forging together steel and iron, forming the celebrated *damasked* steel, which is used for sword blades, springs, etc., the damask appearance of which is produced by a diluted acid, which gives a black tint to the steel while the iron remains white.

Various fancy steels, or alloys of steel with silver, platinum, rhodium and aluminum, have been made with a view to imitating the Damascus steel, wootz, etc., and improving the fabrication of some of the finer kinds of surgical and other instruments.

Properties of Steel.—After being tempered it is not easily broken; it welds readily; it does not crack or split; it bears a very high heat, and preserves the capability of hardening after repeated working.

Hardening and Tempering.—Upon these operations the quality of manufactured steel in a great measure depends.

Hardening is effected by heating the steel to a cherry red, or until the scales of oxide are loosened on the surface, and plunging it into a liquid, or placing it in contact with some cooling substance. The degree of hardness depends upon the heat and the rapidity of cooling. Steel is thus rendered so hard as to resist the hardest files, and it becomes at the same time extremely brittle. The degree of heat and the temperature and nature of the cooling medium must be chosen with reference to the quality of the steel and the purpose for which it is intended. Cold water gives a greater hardness than oils or other fatty substances, sand, wet iron scales or cinders, but an inferior degree of hardness to that given by acids. Oil, tallow, etc., prevent the cracks which are caused by too rapid cooling. The lower the heat at which the steel becomes hard the better.

Tempering.—Steel, in its hardest state, being too brittle for most purposes, the requisite strength and elasticity are obtained by tempering—or *letting down the temper*, as it is termed—which is performed by heating the hardened steel to a certain degree and cooling it quickly. The requisite heat is usually ascertained by the color which the surface of the steel assumes from the film of oxide thus formed. The degrees of heat to which these several colors correspond are as follows :

At 430°, a very faint yellow At 450°, a pale straw color.	Suitable for hard instruments ; as hammer faces, drills, etc.
At 470°, a full yellow...... At 490°, a brown color.....	For instruments requiring hard edges without elasticity ; as shears, scissors, turning tools, etc.
At 510°, brown, with purple spots.................... At 538°, purple............	For tools for cutting wood and soft metals ; such as plane irons, knives, etc.
At 550°, dark blue......... At 560°, full blue..........	For tools requiring strong edges without extreme hardness ; as cold chisels, axes, cutlery, etc.
At 600°, grayish blue, verging on black..............	For spring temper, which will bend before breaking ; as saws, sword blades, etc.

If the steel is heated higher than this the effect of the hardening process is destroyed.

Case-hardening.

This operation consists in converting the surface of wrought iron into steel by cementation, for the purpose of adapting it to receive a polish or to bear friction, etc. This is effected by heating iron to a cherry red in a close vessel, in contact with carbonaceous materials, and then plunging it into cold water. Bones, leather, hoofs, and horns of animals are generally used for this purpose, after having been burned or roasted so that they can be pulverized. Soot is also frequently used.

LIMES, CEMENTS AND MORTARS.

Limestones.

The calcination of marble, or any pure limestone, produces lime (*quick-lime*). The pure limestones burn white and give the richest limes.

The finest calcareous minerals are the rhombohedral prisms of calcareous spar, the transparent double reflecting Iceland spar, and white or statuary marble.

The property of hardening under water, or when excluded from air, conferred upon a paste of lime, is effected by the presence of foreign substances—as silicum, alumina, iron, etc., when their aggregate presence amounts to one tenth of the whole.

Limes are classed: 1. The common or fat limes. 2. The poor or meagre. 3. The hydraulic. 4. The hydraulic cements. 5. The natural pozzuolanas, including pozzuolana, properly so called, trass or terras, the arènes, ochreous earths, basaltic sands, and a variety of similar substances.

Rich limes are fully dissolved in water frequently renewed, and they remain a long time without hardening; they also increase greatly in volume—from two to three and a half times their original bulks—and will not harden without the action of the air. They are rendered hydraulic by the admixture of pozzuolana or trass.

Rich, fat, or common limes usually contain less than 10 per cent. of impurities.

Hydraulic limestones are those which contain iron and clay, so as to enable them to produce cements which become solid when under water.

The pastes of fat limes shrink, in hardening, to such a degree that they cannot be used as mortar without a large dose of sand.

Poor limes have all the defects of rich limes, and increase but slightly in bulk.

The poorer limes are invariably the basis of the most rapidly setting and most durable cements and mortars, and they are also the only limes which have the property, when in combination with silica, etc., of indurating under water, and are, therefore, applicable for the admixture of hydraulic cements or mortars. Alike to rich limes, they will not harden if in a state of paste under water or in wet soil, or if excluded from contact with the atmosphere or carbonic acid gas. They should be employed for mortar only when it is impracticable to procure common or hydraulic lime or cement, in which case it is recommended to reduce them to powder by grinding.

Lime absorbs, in slaking, a mean of two and a half times its volume, and two and a quarter times its weight of water.

Hydraulic limes are those which readily harden under water. The most valuable or eminently hydraulic set from the second to the fourth day after immersion; at the end of a month they become hard and insoluble, and at the end of six months they are capable of being worked, like the hard, natural limestones. They absorb less water than the pure limes, and only increase in bulk from one and three quarters to two and a half times their original volume.

The inferior grades, or moderately hydraulic, require a longer period, say from fifteen to twenty days' immersion, and continue to harden for a period of six months.

The resistance of nydraulic limes increases if sand is mixed in the propertion of 50 to 180 per cent. of the part in volume; from thence it decreases.

Slaked lime is a hydrate of lime.

M. Vicat declares that lime is rendered hydraulic by the admixture with it of from 33 to 40 per cent. of clay and silica, and that a lime is obtained which does not slake, and which quickly sets under water.

Artificial hydraulic limes do not attain, even under favorable circumstances, the same degree of hardness and power of resistance to compression as the natural limes of the same class.

The close grained and densest limestones furnish the best limes.

Hydraulic limes lose or depreciate in value bv exposure to the air.

Arènes is a species of ochreous sand. It is found in France. On account of the large proportion of clay it contains, sometimes as great as seven tenths, it can be made into a paste with water without any addition of lime; hence, it is sometimes used in that state for walls constructed *en pisé*, as well as for mortar. Mixed with rich lime it gives excellent mortar, which attains great hardness under water, and possesses great hydraulic energy.

Pozzuolana is of volcanic origin. It comprises trass or terras, the arènes, some of the ochreous earths, and the sand of certain graywackes, granites, schists and basalts; their principal elements are silica and alumina, the former preponderating. None contain more than 10 per cent. of lime.

When finely pulverized, without previous calcination, and combined with the paste of fat lime in proportions suitable to supply its deficiency in that element, it possesses hydraulic energy to a valuable degree. It is used in combination with rich lime, and may be made by slightly calcining clay and driving off the water of combination at a temperature of 1200°.

Brick or tile dust combined with rich lime possesses hydraulic energy.

Trass or terras is a blue-black trap, and is also of volcanic origin. It requires to be pulverized and combined with rich lime to render it fit for use, and to develop any of its hydraulic properties.

General Gilmore* designates the varieties of hydraulic limes as follows: "If, after being slaked, they harden under water in periods varying from fifteen to twenty days after immersion, they are slightly hydraulic; if from six to eight days, hydraulic; and if from one to four days, eminently hydraulic."

Pulverized silica burned with rich lime produces hydraulic lime of excellent quality. Hydraulic limes are injured by air slaking in a ratio varying directly with their hydraulicity, and they deteriorate by age.

For foundations in a damp soil or exposure hydraulic limes must be exclusively employed.

Cements.

Hydraulic cements contain a larger proportion of silica, alumina, magnesia, etc., than any of the pre-

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* See his Treatises on Limes, Hydraulic Cements and Mortars, and Papers on Practical Engineering, Engineer Department U. S. A.

ceding varieties of lime. They do not slake after calcination, and are superior to the very best of hydraulic limes, as some of them set under water at a moderate temperature (65°) in from three to four minutes; others require as many hours. They do not shrink in hardening, and make an excellent mortar without any admixture of sand.

Roman cement is made from a lime of a peculiar character, found in England and France, derived from argillo calcarious kidney shaped stones, termed "Septaria."

Rosendale cement is from Rosendale, New York.

Portland cement is made in England and France. It requires less water than the Roman cement, sets slowly, and can be remixed with additional water after an interval of twelve or even twenty-four hours from its first mixture.

The property of setting slowly may be an obstacle to the use of some designations of this cement—as the Boulogne, when required for localities having to contend against immediate causes of destruction—as in sea constructions having to be executed under water and between tides. On the other hand, a quick setting cement is always difficult of use; it requires special workmen and an active supervision.

Artificial cement is made by combining slaked lime with unburned clay in suitable proportions.

Artificial pozzuolana is made by subjecting clay to a slight calcination.

Salt water has a tendency to decompose cements of all kinds.

Mortars.

Lime or cement paste is the cementing substance in mortar, and its proportion should be determined by the rule that *the volume of the cementing sub-*

stance should be somewhat in excess of the volume of voids or spaces in the sand or coarse material to be united, the excess being added to meet imperfect manipulation of the mass.

Hydraulic mortar, if repulverized and formed into a paste after having once set, immediately loses a great portion of its hydraulicity and descends to the level of the moderate hydraulic limes.

All mortars are much improved by being worked or manipulated; and as rich limes gain somewhat by exposure to the air, it is advisable to work mortar in large quantities, and then render it fit for use by a second manipulation.

For an analysis of limestones, etc., etc., see Gen. Gilmore's Treatise, pp. 22–125.

White lime will take a larger proportion of sand than brown lime.

The use of salt water in the composition of mortar injures the adhesion of it.

When a small quantity of water is mixed with slaked lime a stiff paste is made, which, upon becoming dry or hard, has but very little tenacity; but, by being mixed with sand or like substances, it acquires the properties of a cement or mortar.

The proportion of sand that can be incorporated with mortar depends partly upon the degree of fineness of the sand itself, and partly upon the character of the lime. For the rich limes the resistance is increased if the sand is in proportions varying from 50 to 240 per cent. of the paste in volume; beyond this proportion the resistance decreases.

Stone Mortar.—8 parts cement, 3 parts lime and 31 parts of sand.

Brick Mortar.—8 parts cement, 3 parts lime and 27 parts of sand.

Brown Mortar.—Lime, 1 part; sand, 2 parts; and a small quantity of hair.

Lime and sand, and cement and sand, lessen about one third in volume when mixed together.

Calcareous Mortar, being composed of one or more of the varieties of lime or cement, natural or artificial, mixed with sand, will vary in its properties with the quality of the lime or cement used, the nature and quality of the sand, and the method of manipulation.

Mortar.—Lime, 1; clean sharp sand, 2½. An excess of water in slaking the lime swells the mortar, which remains light and porous or shrinks in drying; an excess of sand destroys the cohesive properties of the mass. It is indispensable that the sand should be sharp and clean.

Turkish Plaster, or Hydraulic Cement.—100 lbs. fresh lime reduced to powder, 10 quarts linseed oil and one to two ounces cotton. Manipulate the lime, gradually mixing the oil and cotton, in a wooden vessel, until the mixture becomes of the consistency of bread dough. Dry, and, when required for use, mix with linseed oil to the consistency of paste, and then lay on in coats. Water pipes of clay or metal, joined or coated with it, resist the effect of humidity for very long periods.

Exterior Plaster, or Stucco.—One volume of cement powder to two volumes of dry sand.

In India, to the water for mixing the plaster is added 1 lb. of sugar or molasses to 8 imperial gallons of water for the first coat; and for the second, or finishing, 1 lb. sugar to 2 gallons water.

Powdered slaked lime and smith's forge scales, mixed with blood in suitable proportions, make a moderate hydraulic mortar.